HIGHER MATHS
POLYNOMIALS & QUADRATICS

Master synthetic division, the discriminant, completing the square, and solving polynomial equations with this essential guide.

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1 POLYNOMIALS & DIVISION

Synthetic Division
Use coefficients of the polynomial. If a term is missing (e.g., no x^2), put a 0 in the table.

The Remainder Theorem
When $f(x)$ is divided by $(x - h)$, the remainder is $f(h)$.

The Factor Theorem
If $f(h) = 0$ (Remainder is 0), then $(x - h)$ is a factor.

Solving Equations
Find one factor first, divide to get a quadratic, then factorise the quadratic fully.

2 THE QUADRATIC DISCRIMINANT

The Formula
For $ax^2 + bx + c = 0$, the discriminant is: $b^2 - 4ac$

Two Real Distinct Roots
Condition: $b^2 - 4ac > 0$

Equal Roots (One Repeated Root)
Condition: $b^2 - 4ac = 0$ (Tangent to axis)

No Real Roots
Condition: $b^2 - 4ac < 0$ (Does not cross x-axis)

3 GRAPHS, SHAPES & FORMS

COMPLETING THE SQUARE
Form: $a(x + p)^2 + q$.
Turning point is at $(-p, q)$.

INTERSECTION OF CURVES
Equate the functions: $f(x) = g(x)$.
Rearrange to $= 0$ and solve for x.

FINDING UNKNOWNS
If given 2 pieces of info (e.g., a factor and a remainder), set up simultaneous equations to find a and b.

🎓 SQA EXAM TIPS

- **The "Statement":** For "Show that..." questions, you must write a conclusion: *"Since remainder is 0, (x-k) is a factor."*

- **"Hence":** If part (b) says "Hence", use the quadratic result from your synthetic division in part (a). Do not start from scratch!

- **Inequalities:** When solving $ax^2 + bx + c > 0$, always **sketch the graph** to see where the curve is above or below the axis.