Functions and Sets

1. Determine a possible domain for each of the following functions:

a. $f(x) = \sqrt{x-9}$ b. $f(x) = \frac{9}{x-2}$ c. $f(x) = \frac{6}{(x-5)(x+4)}$ d. $f(x) = \frac{8}{\sqrt{x+5}}$

2. Functions $f(x) = 4x - 1$ and $g(x) = 3x + 5$ are defined on suitable domains. Find an expression for $h(x)$ where:

a. $h(x) = f\big(g(x)\big)$ b. $h(x) = g\big(f(x)\big)$ c. $h(x) = f\big(f(x)\big)$ d. $h(x) = g\big(g(x)\big)$

3. Functions $f(x) = x^2 + 2x - 1$ and $g(x) = x + 5$ are defined on suitable domains. Find an expression for $h(x)$ where:

a. $h(x) = f\big(g(x)\big)$ b. $h(x) = g\big(f(x)\big)$ c. $h(x) = f\big(f(x)\big)$ d. $h(x) = g\big(g(x)\big)$

4. Functions $f(x) = \frac{4}{x-1}$ and $g(x) = 2x + 3$ are defined on suitable domains. Find an expression for $h(x)$ as a single fraction where:

a. $h(x) = f\big(g(x)\big)$ b. $h(x) = g\big(f(x)\big)$ c. $h(x) = f\big(f(x)\big)$ d. $h(x) = g\big(g(x)\big)$

5. The function $f(x)$ is defined on a suiatble domain. Find an expression for $f^{-1}(x)$ where

a. $f(x) = 3x - 4$ b. $f(x) = 4x + 6$ c. $f(x) = \frac{2x+5}{3}$ d. $f(x) = \sqrt[3]{3x-1}$

6. Explain the relation between $f(x)$ and $g(x)$ where $f(x) \times g(x) = -1$.

7. [2006, Paper 1] Two functions, f and g, are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$.

a. Find expressions for i) $f\big(g(x)\big)$ and ii) $g\big(f(x)\big)$

b. Determine the range of values of $y = f\big(x(g)\big) \times g\big(f(x)\big)$

8. Two functions, f and g, are defined by $f(x) = \frac{3}{x+3}$ and $g(x) = 2x - 3$.

a. Write an expression for $h(x)$ where $h(x) = f\big(g(x)\big)$.

b. Determine the domain for the function $h(x)$.

9. Two functions, f and g, are defined by $f(x) = 3x + 5$ and $g(x) = \frac{3}{2x+1}$

a. Write an expression for $h(x)$ as a single fraction where $h(x) = f\big(g(x)\big)$.

b. Another function $r(x) = \frac{3}{2x}$. Write an expression for $p(x)$ where $p(x) = r\big(h(x)\big)$.