6. Trig Equations

Section A - Revision Section

This section will help you revise previous learning which is required in this topic.

R1 Revision of solving basic Trig Equations

Solve the equations:

1. $5\tan x° - 6 = 2, \quad 0 \leq x \leq 360.$

2. $7\sin x° + 1 = -5, \quad 0 \leq x \leq 360$

3. $4\cos x° + 3 = 0, \quad 0 \leq x \leq 360.$

4. $3\tan x° + 3 = 7, \quad 0 \leq x \leq 360.$

5. $4\sin x° - 2 = -3, \quad 0 \leq x \leq 360.$

6. $9\cos x° - 5 = 0, \quad 0 \leq x \leq 360.$

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Trigonometry 1 (Expressions and Functions 1.2)

1. Solve

$2sin2x^{\circ} = \sqrt{3}$, for $0 \leq x \leq 180.$

2. Solve

$\sqrt{2}cos2x^{\circ} = 1$, for $0 \leq x \leq 180.$

3. Solve the equation $sin2x^{\circ} - cosx^{\circ} = 0$ in the interval $0 \leq x \leq 180.$

Trig Equations

4. Solve the equation $3sin2x^\circ = 2sinx^\circ$ for $0 \le x \le 180$.

5. Given that $2cosx^\circ + 5sinx^\circ = \sqrt{29}\cos(x - 68 \cdot 2)^\circ$,

solve $2cosx^\circ + 5sinx^\circ = 0 \cdot 5$, for $0 < x < 360$.

6. Given that $5cosx^\circ + sinx^\circ = \sqrt{26}\cos(x - 11 \cdot 3)^\circ$,

solve $5cosx^\circ + sinx^\circ = 2$, for $0 < x < 360$.

Section C - Operational Skills Section

This section provides problems with the operational skills associated with Exponentials and Logs

O1 ***Basic Trig Equations (including radians)***

1. Solve the equations:

(a) $9\tan 2x^\circ - 5 = 3, \quad 0 \le x \le 180.$

(b) $4\sin 3x^\circ + 1 = -2, \quad 0 \le x \le 360.$

(c) $3\cos 2x^\circ + 2 = 0, \quad 0 \le x \le 360.$

2. Solve the equations:

(a) $\tan(x + 30)^\circ = 3, \quad 0 \le x \le 360.$

(b) $5\sin(x + 10)^\circ + 3 = -1, \quad 0 \le x \le 360.$

(c) $4\cos(x + 26)^\circ + 3 = 0, \quad 0 \le x \le 360.$

(d) $\sqrt{3}\tan\left(x + \frac{\pi}{5}\right) + 1 = 0, \quad 0 \le x \le 2\pi.$

(e) $6\sin(x + 2) - 2 = 1, \quad 0 \le x \le 2\pi.$

(f) $\sqrt{2}\cos\left(x + \frac{\pi}{6}\right) + 1 = 0, \quad 0 \le x \le 2\pi.$

Trig Equations

O2 ***Trig Equations which require a substitution.***

1. Solve the equation $\sin 2x° - \cos x° = 0$, in the interval $0 \leq x < 180$.

2. Solve the equation $\sin x° - \sin 2x° = 0$, in the interval $0 \leq x < 360$.

3. Solve the equation $3\cos 2x + 10\cos x - 1 = 0$, in the interval $0 \leq x < 2\pi$.

4. Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$, in the interval $0 \leq x < 360$.

5. Solve the equation $2\cos 2x - 5\cos x - 4 = 0$, in the interval $0 \leq x < 2\pi$.

6. Solve the equation $\tan^2 x = 3$, in the interval $0 \leq x < \pi$.

7. Solve the equation $\sin\theta = 4\cos\theta$, in the interval $0 \leq x < 2\pi$.

8. **(a)** Express $3\sin x + 4\cos x$ in the form $k\sin(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

(b) Hence solve the equation $3\sin x + 4\cos x - 3 = 0$ in the interval $0 \leq x < 2\pi$.

9. **(a)** Express $5\sin x° + 3\cos x°$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Hence solve the equation $5\sin x + 3\cos x = 4$ in the interval $0 \leq x < 360$.

10. Two curves have equations $y = 6\cos x°$ and $y = \sin 2x°$.

Find the coordinates of the points of intersection in the range $0 \leq x < 360$.

11. Two curves have equations $y = -3\cos 2x°$ and $y = \cos x° + 1$.

Find the coordinates of the points of intersection in the range $0 \leq x < 180$.

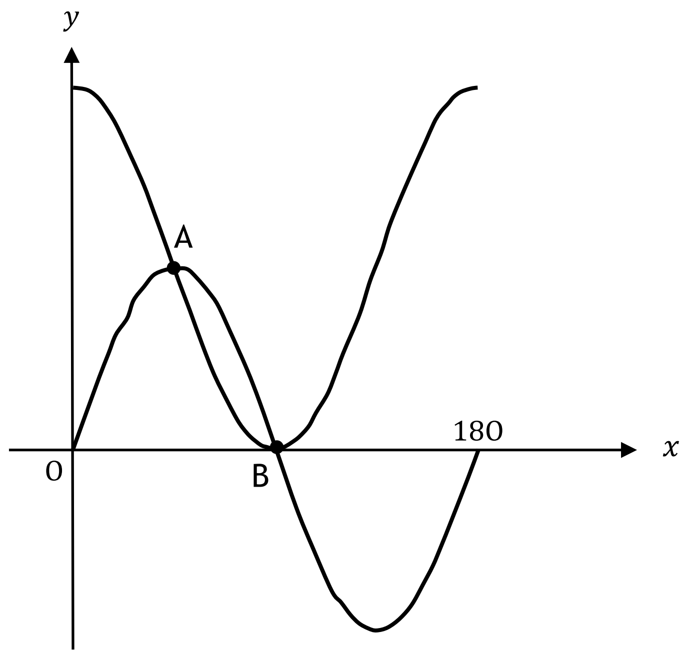
12. A curves has the equation $= \cos 2x° - 3\cos x° + 2$.

Find the coordinates of the points where the curve cuts the x-axis in the range $0 \leq x < 360$.

13. A curves has the equation $= \sin 2x° + \cos x°$.

Find the coordinates of the points where the curve cuts the x-axis in the range $0 \leq x < 360$.

14. The graph shows two curves which have equations $y = 2\cos^2 x°$ and $y = \sin 2x°$ in the range $0 \leq x < 180$.



Find the coordinates of A and B, the points of intersection between the two curves.

O3 ***Trig Equations involving*** **sin and cos *which can be solved by resolving to a*** **tan *equation.***

1. Solve the equations

(a) $\sin x° = \cos x°$, $0 < x < 360$

(b) $2\sin x° - \cos x° = 0,$ $0 < x < 360$

(c) $\sin x° + 5\cos x° = 0,$ $0 < x < 360$

2. Solve the equation $\sin 2x° = 2\cos^2 x°,$ $0 < x < 360$

Section D - Cross Topic Questions

Trigonometry, functions and graphs

1. A function f is defined as $f(x) = \sqrt{3}\cos x° + \sin x°$.

(a) Express $f(x)$ in the form $k\cos(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Sketch the graph of $y = f(x)$ between $0 \leq x < 360$, showing clearly the coordinates of the maximum and minimum turning points.

2. **(a)** Express $3\sin x° + 4\cos x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Sketch the graph of $y = 3\sin x° + 4\cos x° + 1$ between $0 \leq x < 360$, showing clearly the coordinates of the maximum and minimum turning points and where the curve cuts the axes.

Trig Equations

3. Functions $a(x) = \sin x$, $b(x) = \cos x$ and $c(x) = x - \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for;

(i) $a(c(x))$;

(ii) $b(c(x))$.

(b) **(i)** Show that $a(c(x)) = \frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $b(c(x))$ and hence solve the equation $a(c(x)) + b(c(x)) = 1$ for $0 \leq x \leq 2\pi$.

4. Functions f and g are defined on suitable domains by $f(x) = \sin x°$ and $g(x) = 2x$.

(a) Find expressions for;

(i) $f(g(x))$;

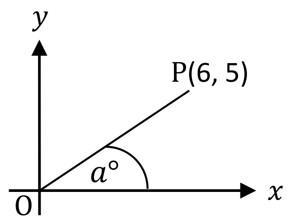
(ii) $g(f(x))$.

(b) Solve $3f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$.

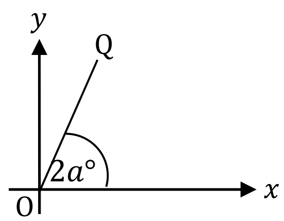
Trigonometry and straight line

5. P is the point $(6, 5)$. The line OP is inclined at an angle of $a°$ to the x-axis.

(a) Find the exact values of $\sin 2a°$ and $\cos 2a°$.



(b) The line OQ is inclined at an angle of $2a°$ to the x-axis. Write down the exact value of the gradient of OQ.



Trig Equations

Answers

Section A

R1

1. $x = 58, 238$ **2.** $x = 239, 301$ **3.** $x = 139, 221$

4. $x = 53 \cdot 1, 233$ **5.** $x = 194 \cdot 5, 345 \cdot 5$ **6.** $x = 56 \cdot 3, 304$

Section B

1. $x^\circ = 30^\circ$ *and* 60° **2.** $x^\circ = 22 \cdot 5^\circ$ *and* $157 \cdot 5^\circ$

3. $x^\circ = 30^\circ, 90^\circ$ *and* 150° **4.** $x^\circ = 0^\circ, 70 \cdot 5^\circ$ *and* 180°

5. $x^\circ = 152 \cdot 9^\circ$ *and* $343 \cdot 5^\circ$ **6.** $x^\circ = 78 \cdot 2^\circ$ *and* $304 \cdot 4^\circ$

Section C

O1

1. **(a)** $x = 20 \cdot 8, 110.8$ **(b)** $x = 76 \cdot 2, 103.8, 196.2, 223.8, 316.2, 343.8$

(c) $x = 65 \cdot 9, 114.1, 245.9, 294.1$

2. **(a)** $x = 41 \cdot 6, 221.6$ **(b)** $x = 223, 297$ **(c)** $x = 112.6, 195.4$

(d) $x = \frac{19\pi}{30}, \frac{49\pi}{30}$ **(e)** $x = 0 \cdot 62, 4 \cdot 8$ **(f)** $x = \frac{7\pi}{12}, \frac{13\pi}{12}$

O2

1. $x = 30, 90, 150$ **2.** $x = 0, 60, 180, 300$ **3.** $x = 1 \cdot 23, 5 \cdot 05$

4. $x = 90, 199 \cdot 5, 340 \cdot 5$ **5.** $x = 2 \cdot 42, 3 \cdot 86$ **6.** $x = \frac{\pi}{3}, \frac{2\pi}{3}$

7. $\theta = 1 \cdot 33, 4 \cdot 47$ **8(a)** $5\sin(x + 0 \cdot 93)$ **(b)** $x = 1 \cdot 57, 6$

9(a) $\sqrt{34}\cos(x - 59)^\circ$ **(b)** $x = 12 \cdot 3, 105 \cdot 7$

10. $(90, 0); (270,0)$ **11.** $\left(60, \frac{3}{2}\right); \left(131 \cdot 8, \frac{1}{2}\right)$

12. $(0, 0); (60,0); (300, 0)$ **13.** $(90, 0); (210,0); (270, 0); (330, 0)$

14. $A(45, 1)$ and $B(90, 0)$

Trig Equations

O3

1. **(a)** 45°,225° **(b)** 26·6°, 206·6° **(c)** 101·3°, 281·3°

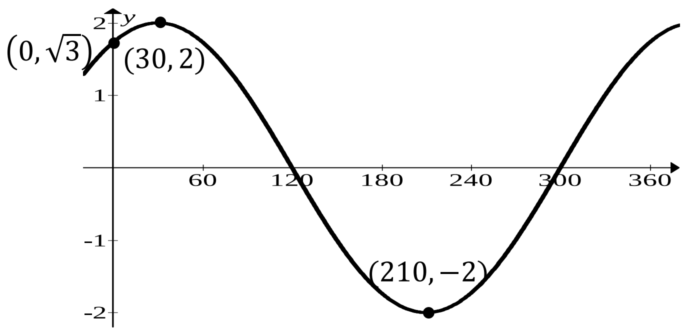
2. 45°, 90°, 225°, 270°

Section D

Trigonometry and functions and graphs

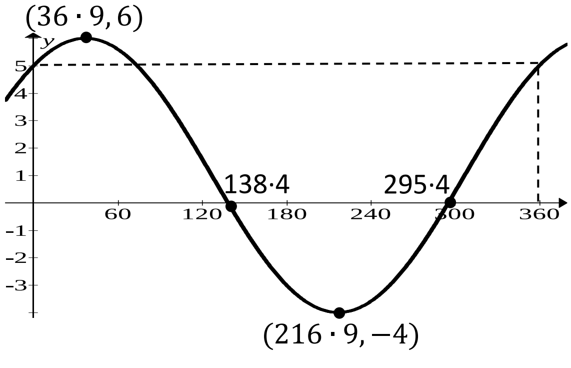
1(a) $2\cos(x-30)°$

(b)



2(a). $5\sin(x+53\cdot1)°$

(b)



3(a). **(i)** $\sin\left(x-\frac{\pi}{4}\right)$ **(ii)** $\cos\left(x-\frac{\pi}{4}\right)$ **(b)** **(i)** Proof **(ii)** $x=\frac{\pi}{4},\frac{3\pi}{4}$

4(a). **(i)** $\sin 2x$ **(ii)** $2\sin x$ **(b)** $x=0,\ 70\cdot5,\ 180,\ 289\cdot5,\ 360$

Trigonometry and the straight line

5(a) $\sin 2a=\frac{60}{61},\ \cos 2a=\frac{11}{61}$ **(b)** $\tan 2a=\frac{60}{11}$