EF3. Functions and Graphs

Section A - Revision Section

This section will help you revise previous learning which is required in this topic.

R1 I have investigated x and y - intercepts for a range of graphs of functions.

Find out where the following graphs cross the x-axis and the y-axis:

(a) $y = 4x + 8$ **(b)** $y = \frac{1}{4}x - 3$ **(c)** $3x + 5y - 15 = 0$

(d) $y = x^2 - 3x$ **(e)** $y = x^2 - 16$ **(f)** $y = x^2 + 6x - 27$

(g) $y = 2x^2 - 18$ **(h)** $y = 2x^2 + 5x - 3$

R2 I can complete the square for a quadratic with coefficient of $x^2 = \pm 1$.

(a) $x^2 + 2x + 5$ **(b)** $t^2 - 10t + 2$ **(c)** $v^2 - 2v + 7$

(d) $7 - 2x - x^2$ **(e)** $1 - 4\mathrm{t} - \mathrm{t}^2$ **(f)** $1 + 2x - x^2$

R3 I have had experience of graphing linear and quadratic functions.

1. Sketch the graphs of the following straight lines:

(a) $y = 2x + 3$ **(b)** $y = -3x - 2$

(c) $y = \frac{1}{2}x + 1$ **(d)** $2x + y - 4 = 0$

2. For the following Quadratic Functions:

- Calculate where the graph crosses the x-axis and the y-axis
- Find the Turning Point and state it’s nature
- Sketch the graph

(a) $y = x^2 - 4x + 3$ **(b)** $y = x^2 - 4x - 12$

Functions and graphs

3. For the following Quadratic Functions:

- Express in the form $y = a(x + b)^2 + c$
- Find the Turning Point, and state it's nature, and find where the graph cuts the y-axis.

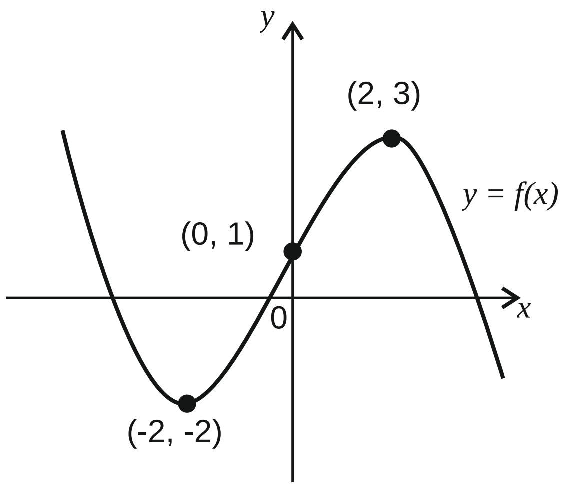
(a) $y = x^2 + 6x - 1$ **(b)** $y = x^2 - 4x + 5$

(c) $y = x^2 + 3x + 4$ **(d)** $y = x^2 - 5x - 5$

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test (Expressions and Functions 1.3)

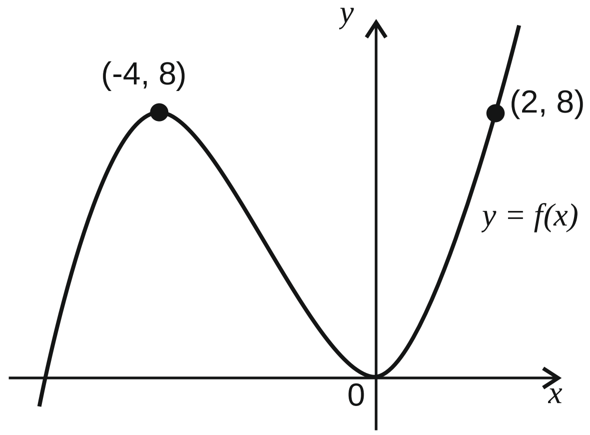
1. The diagram shows the graph of $y = f(x)$ with a minimum turning point at $(-2, -2)$ and a maximum turning point at $(2, 3)$.



Sketch the graph of $y = f(x - 3) + 2$.

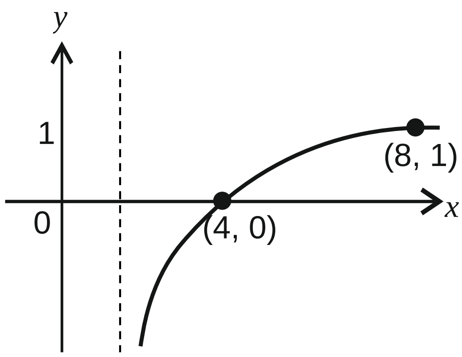
Functions and graphs

2. The diagram shows the graph of $y = f(x)$ with a maximum turning point at $(-4, 8)$ and a minimum turning point at $(0, 0)$.



Sketch the graph of $y = f(x + 2) - 3$.

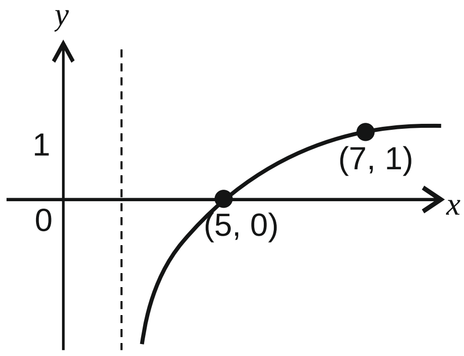
3. The diagram shows the graph of $y = log_b(x - a)$



Determine the values of a and b.

Functions and graphs

4. The diagram shows the graph of $y = log_b(x + a)$

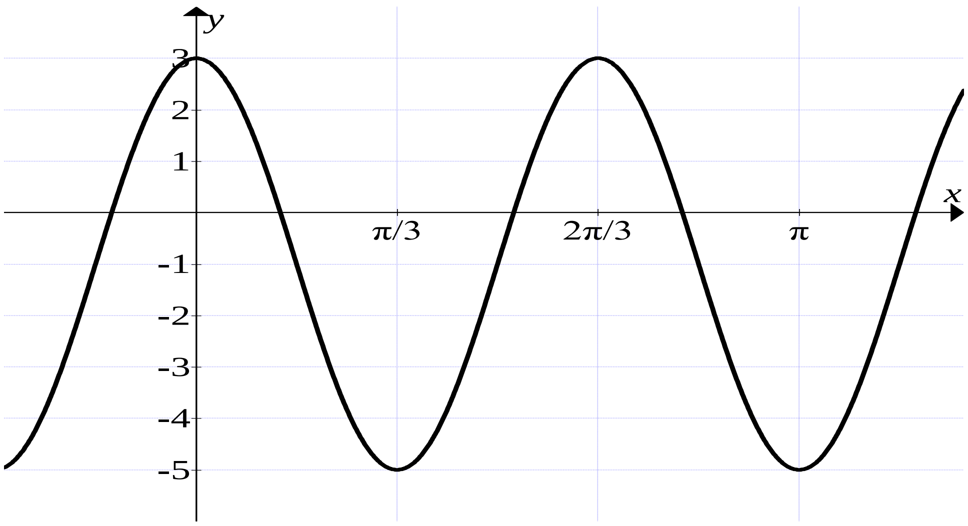


Determine the values of a and b.

5. Sketch the graph of $y = acos(x - \frac{\pi}{3})$ for $0 \leq x \leq 2\pi$ and $a > 0$, clearly showing the maximum and minimum values and where it cuts the x-axis.

6. Sketch the graph of $y = asin(x - \frac{\pi}{6})$ for $0 \leq x \leq 2\pi$ and $a > 0$, clearly showing the maximum and minimum values and where it cuts the x-axis.

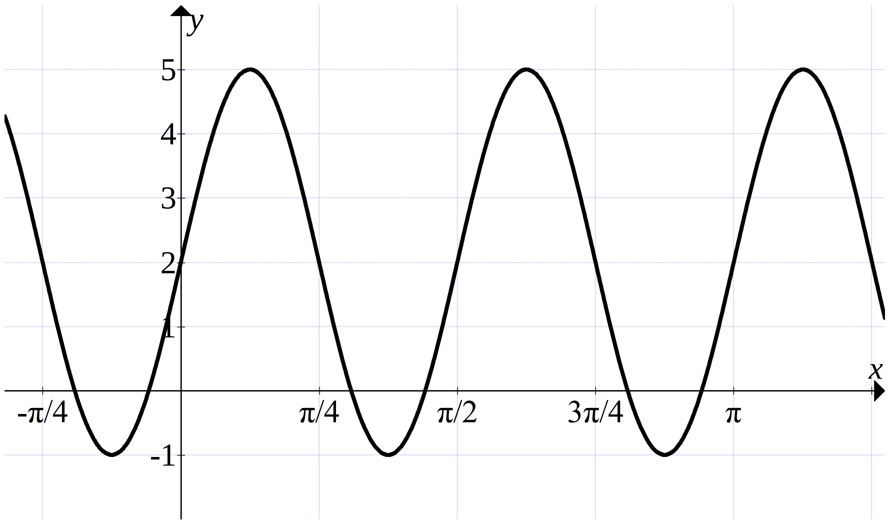
7. The diagram below shows the graph of $y = \mathrm{a}\cos(bx) + c$.



Write down the values of a, b and c.

Functions and graphs

8. The diagram below shows the graph of $y = a\sin(bx) + c$.



Write down the values of a, b and c.

9. The functions f and g, defined on suitable domains, are given by $f(x) = 2x + 3$ and $g(x) = \frac{x^2+25}{x^2-25}$ where $x \neq \pm 5$.

A third function $h(x)$ is defined as $h(x) = g(f(x))$.

(a) Find an expression for $h(x)$.

(b) For which real values of x is the function $h(x)$ undefined?

10. A function is given by $f(x) = 3x^2 + 1$. Find the inverse function $f^{-1}(x)$.

Functions and graphs

Section C – Operational Skills Section

This section provides problems with the operational skills associated with Functions and Graphs.

O1 I can understand and use basic set notation.

1. Using the { } brackets notation, list the following sets:

(a) The set of the first ten prime numbers.

(b) The set of odd numbers greater than 20 but less than 30.

2. Describe the following sets in words:

(a) { Cone, Pyramid }

(b) { 1, 4, 9, 16, 25 }

3. Connect these numbers with the appropriate set, using $\in$:

Numbers: -3 , 0 , $-\frac{2}{5}$, 7

Sets: N, W, Z , Q

4. State which of the following are true and which are false:

(a) 2 $\in$ { prime numbers }

(b) { 0 } is the empty set

(c) { k,l,m,n } = { m,l,k,n }

(d) If A = { whole numbers greater than 50 }, then 46 $\notin$ A

Functions and graphs

5. Using set notation, rewrite the following:

(a) 3 is a member of the set W.

(b) The empty set.

(c) x does not belong to the set A.

(d) S is a subset of the set T.

(e) The set P is equal to the set Q.

6. $S = \{ 1,2,3,4,5,6,7,8,9,10 \}$. List the following subsets of S:

(a) The set of prime numbers in S.

(b) The set of elements in S which are factors of 70.

7. Find a set equal to each of the following:

(a) $\{ 1,2,3 \} \cap \{ 2,3,4,5 \}$

(b) $\{ 1,2,3 \} \cap \{ 3,1,2 \}$

(c) $\emptyset \cap \{ 2,3,4,5 \}$

8. $E = \{ 1,2,3,4,5,6,8,10 \}$ $A = \{ 1,2,3,4 \}$ $B = \{ 3,4,5 \}$ and $C = \{ 2,4,6,8,10 \}$

(a) Find $A \cap B$, $B \cap C$ and $A \cap C$.

(b) The set of elements common to A,B and C is denoted by $A \cap B \cap C$.

Find $A \cap B \cap C$.

9. Given that $A = \{ 0,1,2 \}$, which of the following are true?

(a) $2 \in A$ **(b)** $1 \subset A$ **(c)** $\{1\} \subset A$

(d) $0 \in \emptyset$ **(e)** $A \subset A$ **(f)** $1 \notin A$

Functions and graphs

10. P = { 1,2,3,4,5,6,7 } Q = { 5,6,7,8,9,10 } are subsets of E = { 1,2,3,...,12 }.

List the members of the following sets:

(a) P ∩ Q **(b)** P ∪ Q **(c)** P'

(d) Q' **(e)** (P ∩ Q)' **(f)** (P ∪ Q)'

(g) P ∩ Q' **(h)** P' ∩ Q **(i)** P ∩ ∅

O2 I have investigated domains and ranges.

1. State a suitable domain for the following functions:

(a) $f(x) = \frac{x^2}{x-1}$ **(b)** $f(x) = \frac{4x-2}{2x-3}$ **(c)** $f(x) = \frac{2x+7}{x^2-16}$

(d) $f(x) = \frac{x^2-5x+4}{x^2+8x+12}$ **(e)** $f(x) = \sqrt{10-x}$ **(f)** $f(x) = \sqrt{x^2+3x}$

2. State the range of each function given its domain:

(a) $f(x) = 3x - 4$; $x \in \{2, 3, 4, 5\}$

(b) $f(x) = x^2 - 3x + 4$; $x \in \{-2, -1, 0, 1, 2\}$

O3 I can determine a composite function.

1. Given $f(x) = 2x - 3$, $g(x) = x^2$ and $h(x) = x^2 + 4$, find the following functions:

(a) $f(g(x))$ **(b)** $g(f(x))$ **(c)** $h(f(x))$

(d) $f(f(x))$ **(e)** $g(h(x))$ **(f)** $h(h(x))$

Functions and graphs

2. Given $f(x) = x - 2$, $g(x) = \frac{2}{x^2}$ and $h(x) = \frac{4}{x+1}$, find the following functions:

(a) $h(f(x))$ **(b)** $g(f(x))$ **(c)** $f(h(x))$

(d) $f(g(x))$ **(e)** $g(h(x))$ **(f)** $h(h(x))$

3. Given $f(x) = x + 2$, $g(x) = e^x$ and $h(x) = \tan x$, find the following functions:

(a) $g(f(x))$ **(b)** $g(g(x))$ **(c)** $h(f(x))$

4. Given $f(x) = 3x^2 + 2x - 1$, $g(x) = \sin x$ and $h(x) = \log_4 x$, find the following functions:

(a) $f(g(x))$ **(b)** $h(f(x))$ **(c)** $g(g(x))$

5. Two functions f and g, are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

(a) Find expressions for $f(g(x))$ and $g(f(x))$.

(b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.

6. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$, are defined on a set of real numbers.

(a) Find h(x) where $h(x) = g(f(x))$.

(b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$

(ii) Hence state the range of the function h.

7. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

(a) Find an expression for $h(x)$ where $h(x) = f(g(x))$.

(b) Write down any restriction on the domain of h.

8. Functions $f(x) = \frac{1}{x+2}$ and $g(x) = 3x - 1$ are defined on suitable domains.

(a) Find an expression for $h(x)$ where $h(x) = f(g(x))$.

(b) Write down any restriction on the domain of h.

O4 I understand that $f(g(x)) = x$ implies that $g(x)$ is the inverse of $f(x)$.

1. If $f(x) = 3x - 2$ and $g(x) = \frac{x+2}{3}$

(a) Find $f(g(x))$ and $g(f(x))$.
(b) State a relationship between $f(x)$ and $g(x)$.

2. If $f(x) = 2x + 5$ and $g(x) = \frac{x-5}{2}$

(a) Find $f(g(x))$ and $g(f(x))$.
(b) State a relationship between $f(x)$ and $g(x)$.

Functions and graphs

O5 *I can determine the inverse of a linear function.*

1. Given $g(x) = 5x + 2$, find an expression for $g^{-1}(x)$.

2. Given $h(x) = 2x - 6$, find an expression for $h^{-1}(x)$.

3. Given $g(x) = \frac{1}{4}x - 3$, find an expression for $g^{-1}(x)$.

4. Given $f(x) = 2 - 4x$, find an expression for $f^{-1}(x)$.

5. Given $g(x) = \frac{2x-4}{5}$, find an expression $g^{-1}(x)$.

6. **Given** $g(x) = 6 - 2x$, write down an expression for $g(g^{-1}(x))$.

O6 *I can complete the square for any quadratic and understand the connection to its graph.*

1. **(a)** Show that the function $f(x) = 3x^2 + 30x + 73$ can be written in the form $f(x) = a(x+b)^2 + c$, where a, b and c are constants.

(b) Hence or otherwise find the coordinates of the turning point of function $f(x)$.

2. **(a)** Show the function $f(x) = 9 - 8x - x^2$ can be written in the form $f(x) = p(x+q)^2 + r$ where p, q and r are constants.

(b) Hence or otherwise find the maximum value of $f(x)$.

3. The cost, c pence of running a car for 20 miles at an average speed of x mph is given by $c = \frac{1}{4}x^2 - 25x + 875$

(a) Express c in the form $p(x-q)^2 + r$

(b) Find the most economical average speed and hence the cost for 20 miles at this speed

4. The height h metres, of a toy rocket is given by $h = 60 + 10t - t^2$ where t seconds is the time of flight

(a) Express h in the form $p(t+q)^2 + r$

(b) Find the maximum height of the rocket and the time taken to reach it

5. **(a)** Show that the function $f(x) = 4x^2 + 16x - 5$ can be written in the form $f(x) = a(x+b)^2 + c$, where a, b and c are constants.

(b) Hence or otherwise, find the coordinates of the turning point of the function f.

6. **(a)** Express $f(x) = 10 - 6x - 3x^2$ in the form $f(x) = a(x+b)^2 + c$ where a, b and c are constants.

(b) Find the nature and the coordinates of the turning point of the function.

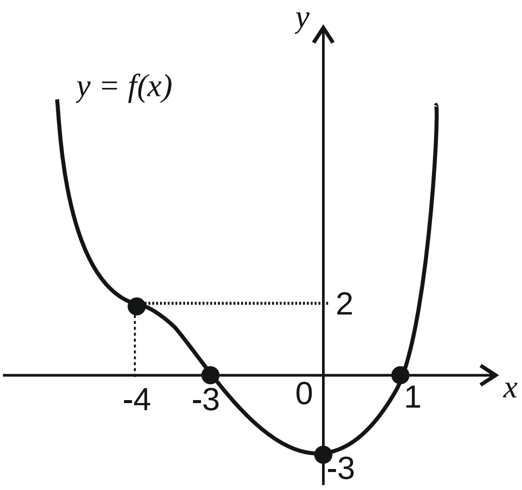
O7 I can identify and sketch a function after a transformation of the form $kf(x), f(x) + k, f(kx), f(x + k), -f(x), f(-x)$, or a combination of these.

1. The diagram shows the graph of a function f.

f has a minimum turning point at (0, –3) and a point of inflexion at (–4, 2).

(a) Sketch the graph $y = f(-x)$.

(b) On the same diagram, sketch the graph $y = 2f(-x)$.

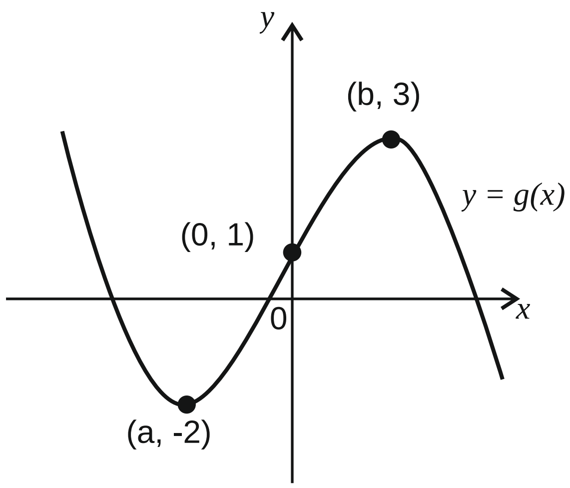


Functions and graphs

2. The diagram shows the graph of $y = g(x)$.

(a) Sketch the graph of $y = -g(x)$.

(b) On the same diagram, sketch the graph $y = 3 - g(x)$.

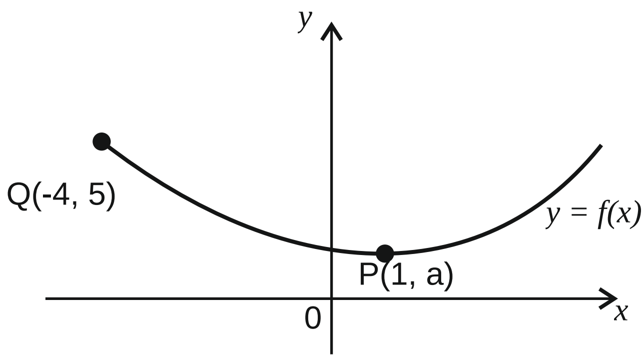


3. The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:

(a) $y = f(x-4)$.

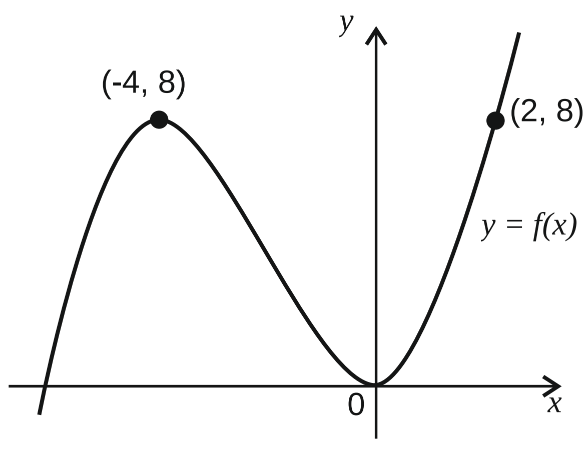
(b) $y = 2 + f(x-4)$.



4. The diagram shows a sketch of the function $y = f(x)$.

(a) Copy the diagram and on it sketch the graph of $y = f(2x)$.

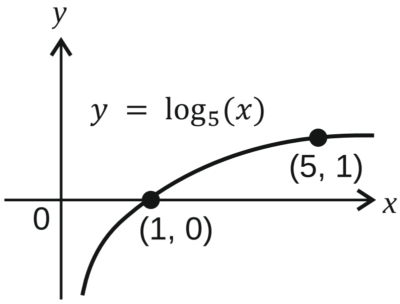
(b) On a separate diagram sketch the graph of $y = 1 - f(2x)$.



Functions and graphs

O8 ***I can sketch logarithmic and exponential functions and determine a suitable domain or range for a given function/composite function.***

1.



The diagram shows a sketch of part of the graph of $y = \log_5 x$.

(a) Make a copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 x + 1$.

Find the coordinates of the point where it crosses the x-axis.

(b) Make a second copy of the graph of $y = \log_5 x$.

On your copy, sketch the graph of $y = \log_5 \frac{1}{x}$.

2. The functions f and g, defined on suitable domains, are given by

$f(x) = \frac{1}{x^2-4}$ and $g(x) = 2x + 1$.

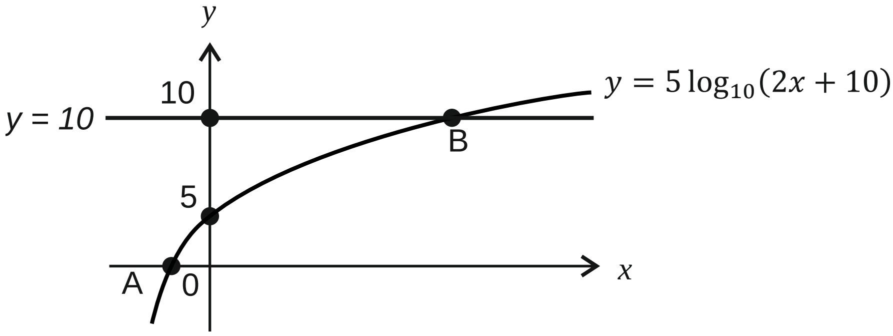
(a) Find an expression for $h(x)$ where $h(x) = g(f(x))$.

Give your answer as a single fraction.

(b) State a suitable domain for h.

Functions and graphs

3. Part of the graph of $y = 5\log_{10}(2x+10)$ is shown in the diagram below.



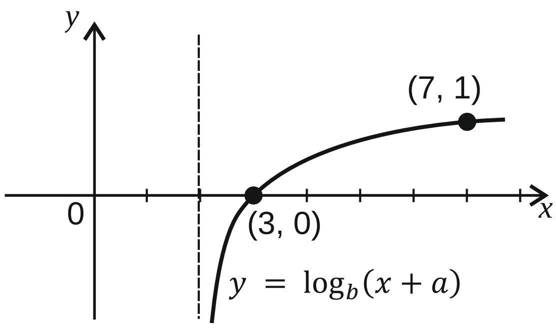
This graph crosses the x-axis at the point A and the straight line $y = 10$ at the point B.

(a) Find the x-coordinate of A.

(b) Find the x-coordinate of B.

4. The diagram shows part of the graph of $y = \log_b(x+a)$.

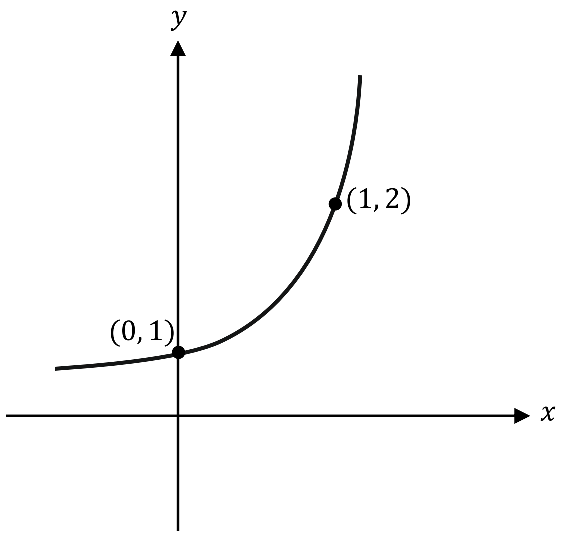
Determine the values of *a* and *b*.



5. The diagram shows part of the graph of $y = 2^x$.

(a) Sketch the graph of $y = 2^{-x} - 8$.

(b) Find the coordinates of the points where it crosses the x and y axes.

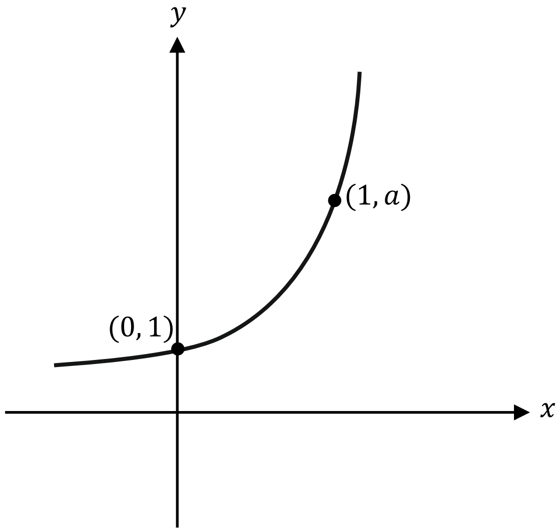


Functions and graphs

6. **(a)** Given $y = a^x$, sketch the graph of $y = a^x + 1,\ a > 2$.

(b) On the same diagram, sketch the graph of $y = a^{x+1},\ a > 2$

a) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$



7. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

(a) Find $h(x)$ where $h(x) = g(f(x))$.

(b) (i) Write down the coordinates of the minimum turning point $y = h(x)$.

(ii) Hence state the range of the function h.

8. Sketch the following pairs of graphs on the same set of axes:

(a) $y = a^x$ and $y = 3(a^x)$

(b) $y = 3^x$ and $y = 3^{(x+1)}$

(c) $\mathrm{y} = \log_2 x$ and $\mathrm{y} = \log_2 4(x-1)$

(d) $\mathrm{y} = \log_4 x$ and $\mathrm{y} = \log_4 x^3$

Functions and graphs

Section D - Cross Topic Exam Style Questions

Functions and Logs

1. Functions f, g and h are defined on suitable domains by

$f(x) = x^2 - x + 10$ $\quad g(x) = 5 - x$ $\quad$ and $\quad h(x) = \log_2 x$

(a) Find expressions for $h(f(x))$ and $h(g(x))$.

(b) Hence solve $h(f(x)) - h(g(x)) = 3$.

Functions and Trig

2. Functions a and b are defined on suitable domains by

$a(x) = x + 30$ and $b(x) = \cos x°$.

Show that $b\big(a(x)\big) = \frac{1}{2}\big(\sqrt{3}\cos x° - \sin x°\big)$.

Functions and graphs

Section A

R1

1. **(a)** $(-2, 0), (0, 8)$ **(b)** $(12, 0), (0, -3)$

(c) $(5, 0), (0, 3)$ **(d)** $(0, 0),\ (3, 0)$

(e) $(-4, 0)(4, 0)(0, -16)$ **(f)** $(-9, 0)(3, 0)(0, -27)$

(g) $(-3, 0)(3, 0)(0, -18)$ **(h)** $(-3, 0)\left(\frac{1}{2}, 0\right)(0, -3)$

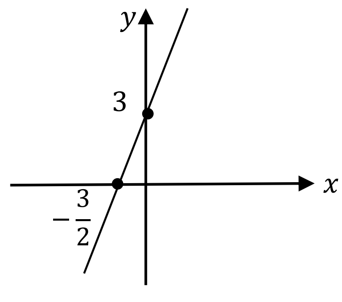
R2

(a) $(x + 1)^2 + 4$ **(b)** $(t - 5)^2 - 23$ **(c)** $(v - 1)^4 + 6$

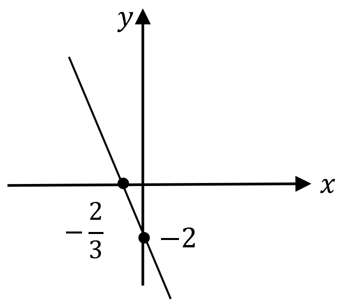
(d) $8 - (x + 1)^2$ **(e)** $5 - (t + 2)^2$ **(f)** $2 - (x - 1)^2$

R3

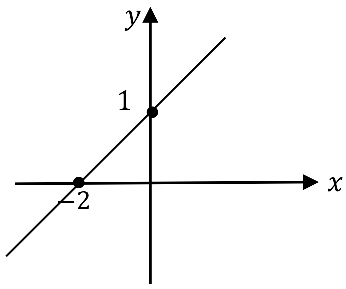
1. **(a)**



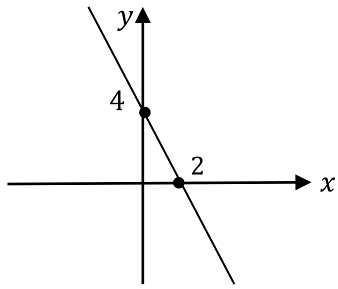
(b)



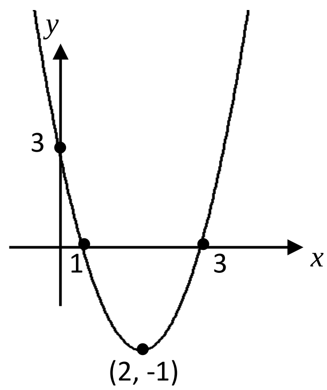
(c)



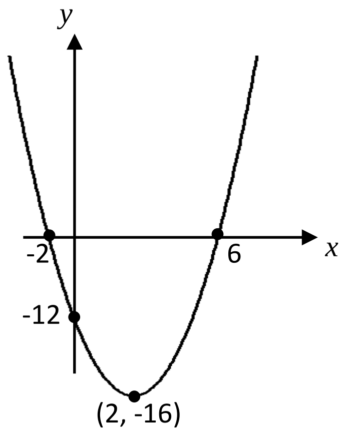
(d)



2. **(a)** $(1, 0), (3, 0), (0, 3); min\ at\ (2, -1)$



(b) $(-2,0),(6,0),(0,-12); min\ at\ (2,-16)$



3. **(a)** $y=(x+3)^2-10;\ min\ at\ (-3,-10); y-intercept\ (0,-1)$

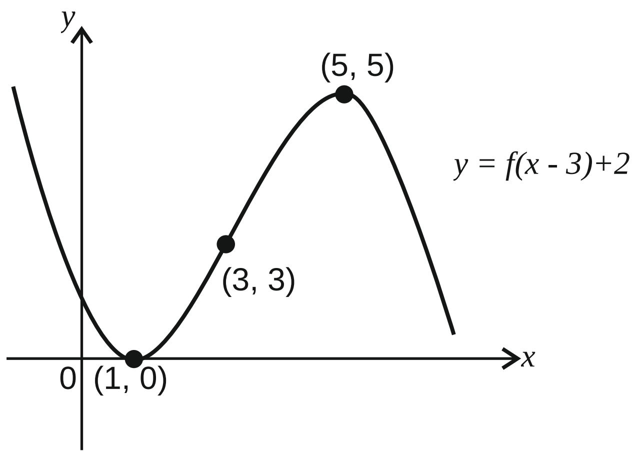
(a) $y=(x-2)^2+1;\ min\ at\ (2,1); y-intercept\ (0,5)$

(a) $y=\left(x+\frac{3}{2}\right)^2+\frac{7}{4};\ min\ at\ \left(-\frac{3}{2},\frac{7}{4}\right); y-intercept\ (0,4)$

(a) $y=\left(x-\frac{5}{2}\right)^2-\frac{45}{4};\ min\ at\ \left(\frac{5}{2},-\frac{45}{4}\right); y-intercept\ (0,-5)$

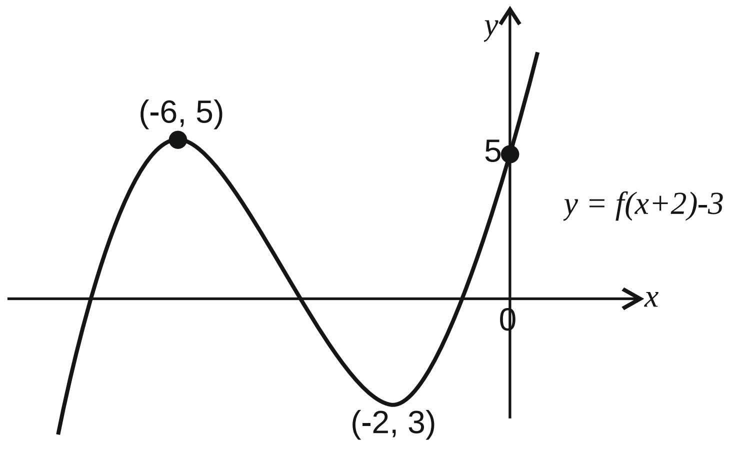
Section B - Assessment Standard Section

1.



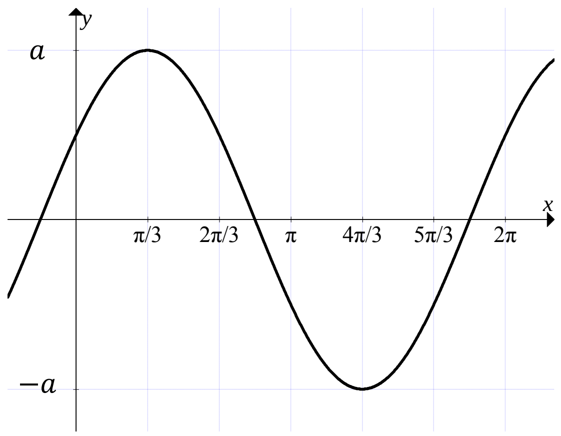
Functions and graphs

2.

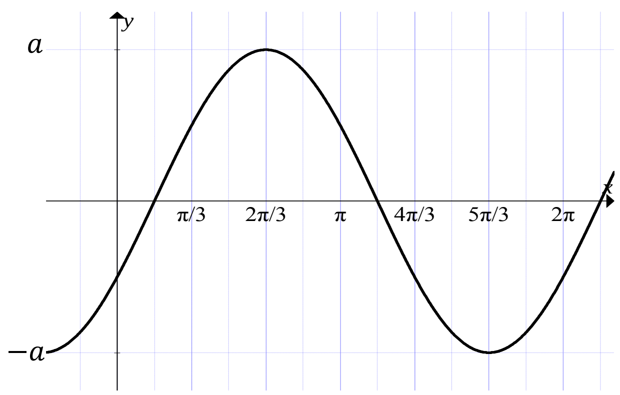


3. $a = 3\ b = 5$ **4.** $a = -4\ b = 3$

5.



6.



7. $a = 4\ b = 3\ c = -1$

Functions and graphs

8. $a = 3\ b = 4\ c = 2$

9. **(a)** $h(x) = \frac{(2x+3)^2+25}{(2x+3)^2-25}$ (b) $h(x)$ undefined for $x = -4\ and\ x = 1.$

10. $f^{-1}(x) = \sqrt{\frac{x-1}{3}}$

Section C

O1

1. **(a)** $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$ **(b)** $\{21, 23, 25, 27, 29\}$

2. **(a)** A set containing two 3D shapes

(b) A set containing the first 5 square numbers

3. $7 \in N, W, Z, Q;$ $-3 \in Z, Q;$ $0 \in W, Z, Q;$ $-\frac{2}{5} \in Q$

4. **(a)** T **(b)** F **(c)** T **(d)** T

5. **(a)** $3 \in W$ **(b)** $\emptyset$ **(c)** $x \notin A$ **(d)** $S \subset T$ **(e)** $P = Q$

6. **(a)** $\{2, 3, 5, 7\}$ **(b)** $\{1, 2, 5, 7, 10, 14, 35, 70\}$

7. **(a)** $\{2, 3\}$ (b) $\{1, 2, 3\}$ **(c)** $\emptyset$

8. **(a)** $\{3,4\}$, $\{4\}$, $\{2, 4\}$ (b) $\{4\}$

9. **(a)** T **(b)** F **(c)** T **(d)** F **(e)** T

(f) F

10. **(a)** $\{5, 6, 7\}$ **(b)** $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **(c)** $\{8, 9, 10, 11, 12\}$

(d) $\{1, 2, 3, 4, 11, 12\}$ **(e)** $\{1, 2, 3, 4, 8, 9, 10, 11, 12\}$

(f) $\{11, 12\}$ **(g)** $\{1, 2, 3, 4\}$

(h) $\{8, 9, 10\}$ **(i)** $\emptyset$

O2

1. **(a)** $\{x: x \in R, x \neq 1\}$ **(b)** $\left\{x: x \in R, x \neq \frac{3}{2}\right\}$ **(c)** $\{x: x \in R, x \neq \pm 4\}$

(d) $\{x: x \in R, x \neq -2, x \neq -6\}$ **(e)** $\{x: x \in R, x \leq 10\}$ **(f)** $\{x: x \in R, x \leq -3, x \geq 0\}$

2. **(a)** $f(x) \in \{2, 5, 8, 11\}$ (b) $f(x) \in \{2,4, 8, 14\}$

Functions and graphs

O3

1. (a) $f(g(x)) = 2x^2 - 3$ (b) $g(f(x)) = (2x-3)^2$

(c) $h(f(x)) = (2x-3)^2 + 4$ (d) $f(g(x)) = 4x - 9$

(e) $g(h(x)) = (x^2+4)^2$ (f) $h(h(x)) = (x^2+4)^2 + 4$

2. (a) $h(f(x)) = \frac{4}{x-1}$ (b) $g(f(x)) = \frac{2}{(x-2)^2}$

(c) $f(h(x)) = \frac{4}{x+1} - 2$ (d) $f(g(x)) = \frac{2}{x^2} - 2$

(e) $g(h(x)) = \frac{(x+1)^2}{8}$ (f) $h(h(x)) == \frac{4x+4}{x+5}$

3. (a) $g(f(x)) = e^{(x+2)}$ (b) $g(g(x)) = e^{e^x}$

(c) $h(f(x)) = \tan(x+2)$

4. (a) $f(g(x)) = 3\sin^2 x + 2\sin x - 1$ (b) $h(f(x)) = \log_4(3x^2 + 2x - 1)$

(c) $g(g(x)) = \sin(\sin x)$

5. (a) $f(g(x)) = 4x - 3,\ g(f(x)) = 4x + 3$ (b) -9

6. (a) $h(x) = 9x^2 - 6x + 8$ (b)i $\left(\frac{1}{3}, 7\right)$ (b)ii $\{h: h \in R, x \geq 7\}$

7. (a) $h(x) = \frac{1}{2x-1}$ (b) $x \neq \frac{1}{2}$

8. (a) $h(x) = \frac{1}{3x+1}$ (b) $x \neq -\frac{1}{3}$

O4

1. (a) $f(g(x)) = g(f(x)) = x$

(b) $f(x)$ and $g(x)$ are inverse functions

2. (a) $f(g(x)) = g(f(x)) = x$

(b) $f(x)$ and $g(x)$ are inverse functions

O5

1. $g^{-1}(x) = \frac{x-2}{5}$ 2. $h^{-1}(x) = \frac{x+6}{2}$ 3. $g^{-1}(x) = 4(x+3)$

4. $f^{-1}(x) = \frac{2-x}{4}$ 5. $g^{-1}(x) = \frac{5x+4}{2}$ 6. $f^{-1}(x) = \frac{6-x}{2}$

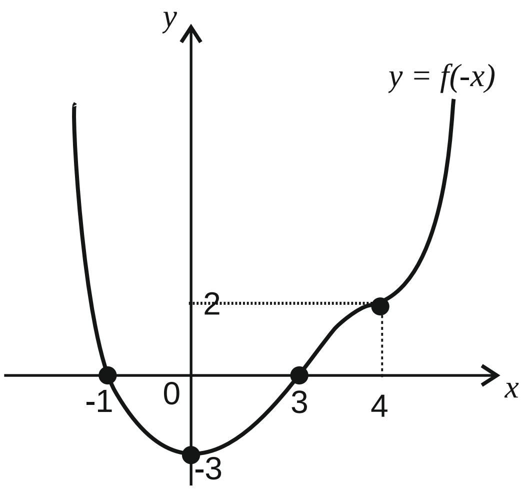
Functions and graphs

O6

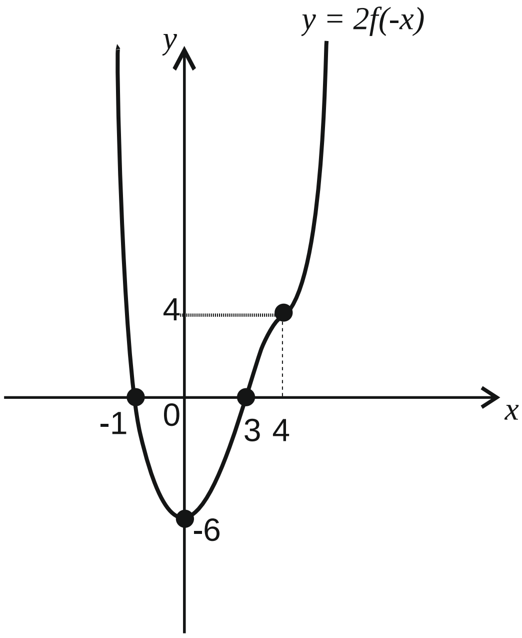
1.	**(a)**	$f(x) = 3(x+5)^2 - 2$	**(b)**	Minimum turning point at $(-5, -2)$
2.	**(a)**	$f(x) = 25 - (x+4)^2$	**(b)**	$f(x)_{max} = 25$
3.	**(a)**	$c = \frac{1}{4}(x-50)^2 + 250$	**(b)**	50 mph with a cost of 250p (£2.50)
4.	**(a)**	$h = 85 - (t-5)^2$	**(b)**	$h_{max} = 85\ metres$ when $t = 5\ secs$
5.	**(a)**	$f(x) = 4(x+2)^2 - 21$	**(b)**	Minimum turning point at $(-2, -21)$
6.	**(a)**	$f(x) = 13 - 3(x+1)^2$	**(b)**	Maximum turning point at$(-1, 13)$

O7

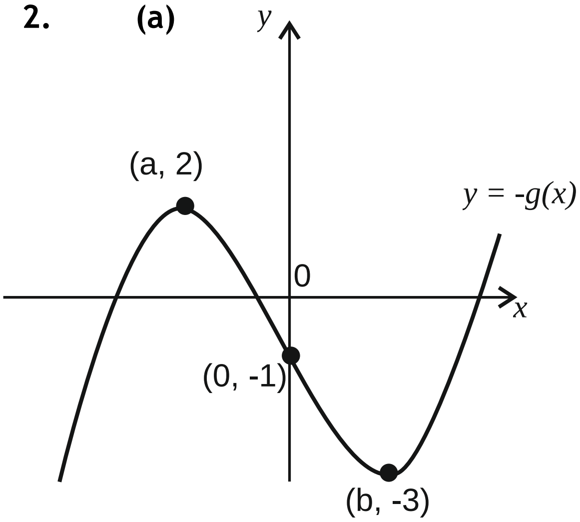
1. **(a)**



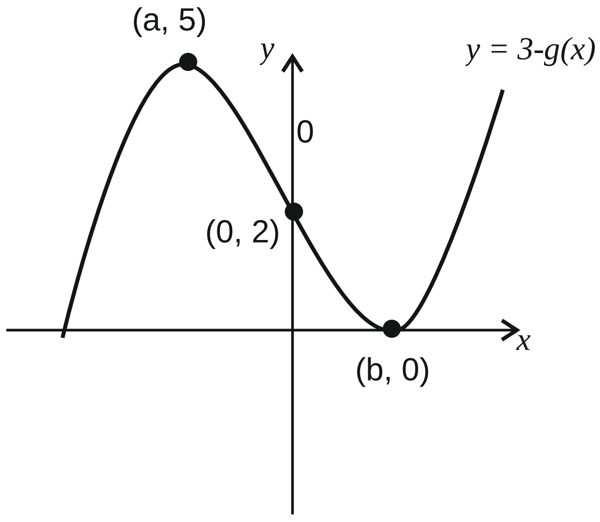
(b)



2. **(a)**

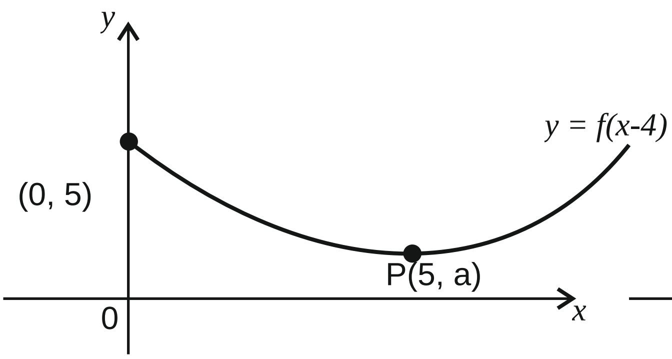


(b)

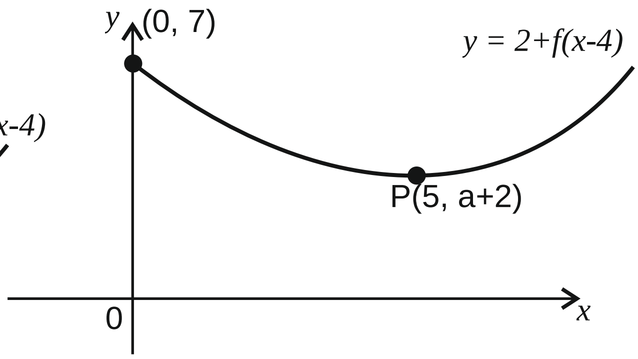


Functions and graphs

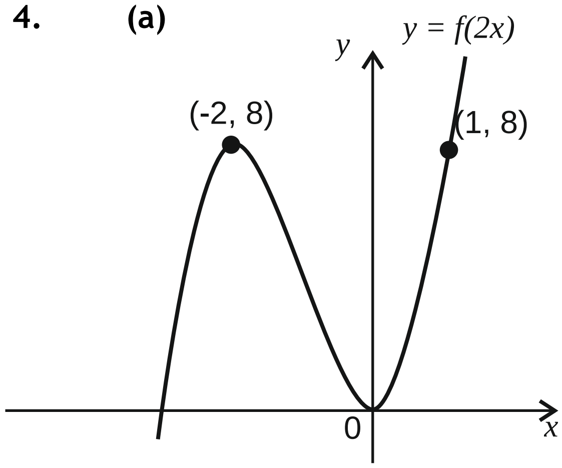
3. **(a)**



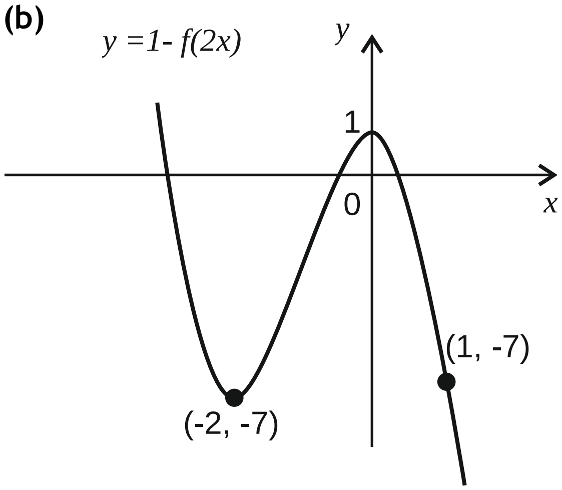
(b)



4. **(a)**



(b)



O8

1. **(a)** Graph translated to pass through $\left(\frac{1}{5}, 0\right)$, $(1, 1)$, $(5, 2)$.

(b) Graph reflected in the x-axis to pass through $(1, 0)$, $(5, -1)$.

2. **(a)** $\frac{x^2-2}{x^2-4}$ (b) $\{x \in R, x \neq \pm 2\}$ **3.** **(a)** $x = -\frac{9}{2}$ **(b)** $x = 45$

4. $a = -2, b = 5$

5. **(a)** Graph reflected in the y-axis then displaced by -8 in the y-direction to pass through $(-1, -6)$, $(0, -7)$.

(b) $(-3,0)$, $(0, -7)$

6. **(a)** (i) Graph translated to pass through $(0, 2)$.

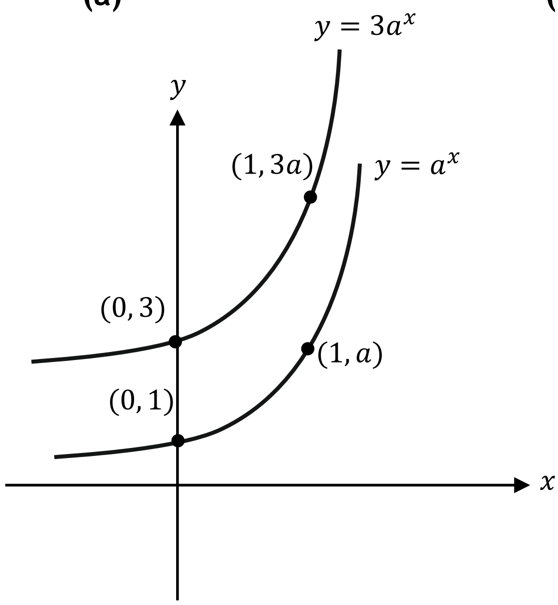
(ii) Graph transformed by a factor of a in the y-direction passing through $(0, a)$.

Functions and graphs

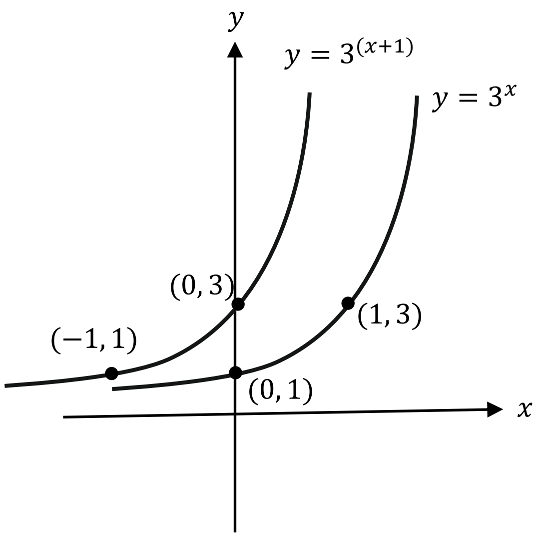
(b) Proof

7. **(a)** $h(x) = 9x^2 - 6x + 8$ **(b)** $\min t.p\ (\frac{1}{3}, 7)$ with$\{x \in R: x > 7\}$

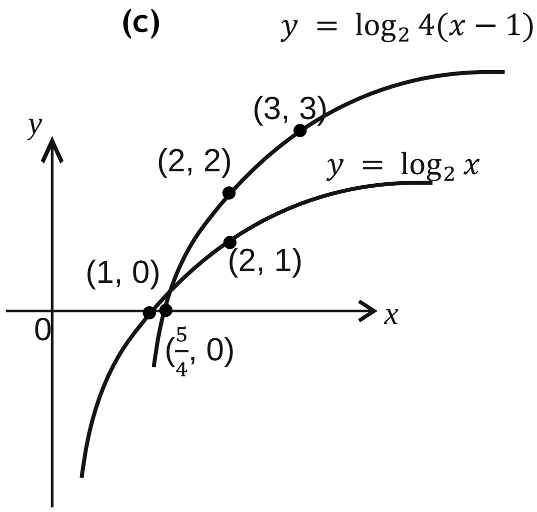
8. **(a)**



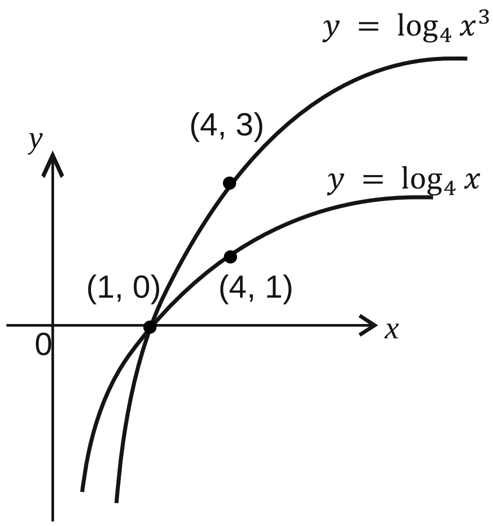
(b)



(c)



(d)



Cross Topic Questions

1. **(a)** $h(f(x)) = \log_2(x^2 - x + 10)$ and $h(g(x)) = \log_2(5 - x)$

(b) $x = -10, 3$

2. Proof.