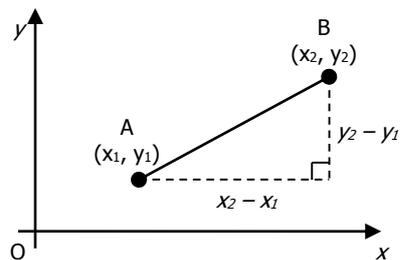# The Straight Line

Revision from National 5

The graph of $y = mx + c$ is a **straight line**, where m is the gradient and c is the y-intercept.

Gradient is a measure of the steepness of a line. The gradient of the line joining points A (x_1, y_1) and B (x_2, y_2) is given by:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Find:

a) the gradient and y-intercept of the line
y = 2x + 5

b) the equation of the line with gradient – 4 and y-intercept (0, -2)

c) the gradient of the line joining P (-2, 4) and Q (3, -1)

d) the gradient of the line 3y + 4x – 11 = 0
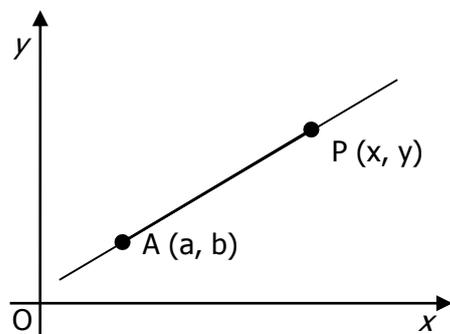
Equation of a Straight Line: $y – b = m(x – a)$

Points A (a, b) and P (x, y) both lie on a straight line.

The gradient of the line $m = \frac{y-b}{x-a}$. Rearranging this gives:

$y – b = m(x – a)$

NOTE: when you are asked to find the equation of a straight line, it is fine to leave it in this form (unless you are specifically asked to remove the brackets).

Example 2: Find the equations of the lines:

a) through (4, 5) with m = 2

b) joining (-1, -2) and (3, 10)

c) parallel to the line x – 2y + 4 = 0 and passing through the point (2, -3)

Equation of a Straight Line
$y = mx + c$ AND $Ax + By + C = 0$

Example 3: Find the equation of the line through (-5, -1) with m = $-\dfrac{2}{3}$, giving your answer in the form $Ax + By + C = 0$.

Example 4: Sketch the line $5x – 2y – 24 = 0$ by finding the points where it crosses the x – and y – axes.
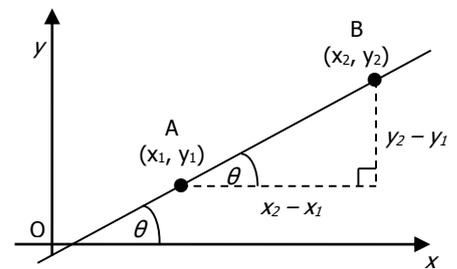

The Angle with the x-axis

The gradient of a line can also be described as the angle it makes with the *positive* direction of the x –axis.

As the y-difference is OPPOSITE the angle and the x-difference is ADJACENT to it, we get:

$\mathbf{m_{AB} = \tan\theta}$
(where θ is measured CLOCKWISE from the x-axis)

Example 5: Find the angle made with the positive direction of the x -axis and the lines:

a) $y = x – 1$

b) $y = 5 – \sqrt{3}x$

c) joining the points (3, -2) and (7, 4)

Gradients of straight lines can be summarised as follows:

a) lines sloping **up** from left to right have **positive** gradients and make **acute** angles with the positive direction of the x-axis
b) lines sloping **down** from left to right have **negative** gradients and make **obtuse** angles with the positive direction of the x-axis
c) lines with **equal** gradients are parallel
d) **horizontal** lines (parallel to the x -axis) have gradient **zero** and equation y = a
e) **vertical** lines (parallel to the y -axis) have gradient **undefined** and equation x = b

If three (or more) points lie on the same line, they are said to be **collinear.**

Example 6: Prove that the points D (-1, 5), E (0, 2) and F (4, -10) are collinear.

If two lines are perpendicular to each other (i.e. they meet at 90°), then:

$$m_1 m_2 = -1$$

Example 7: Show whether these pairs of lines are perpendicular:

a) $x + y + 5 = 0$
    $x - y - 7 = 0$

b) $2x - 3y = 5$
    $3x = 2y + 9$

c) $y = 2x - 5$
    $6y = 10 - 3x$

When asked to find the gradient of a line perpendicular to another, follow these steps:

1. **Find the gradient of the given line**
2. **Flip it upside down**
3. **Change the sign (e.g. negative to positive)**

Example 8: Find the gradients of the lines perpendicular to:
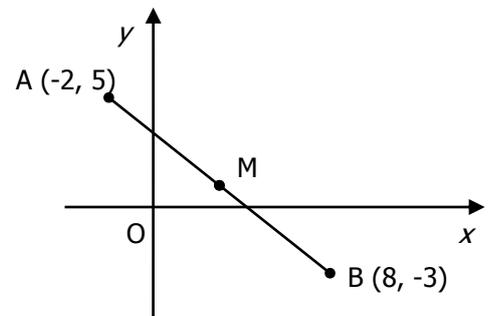
a) the line $y = 3x - 12$

b) a line with gradient = -1.5

c) the line $2y + 5x = 0$

Example 9: Line L has equation $x + 4y + 2 = 0$. Find the equation of the line perpendicular to L which passes through the point (-2, 5).

The **midpoint** of a line lies exactly halfway along it. To find the coordinates of a midpoint, find halfway between the x – and y – coordinates of the points at each end of the line (see diagram).

The x – coordinate of M is halfway between -2 and 8, and its y – coordinate is halfway between 5 and -3.

In general, if M is the midpoint of A (x_1, y_1) and B (x_2, y_2):

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The **perpendicular bisector** of a line passes through its midpoint at 90˚.

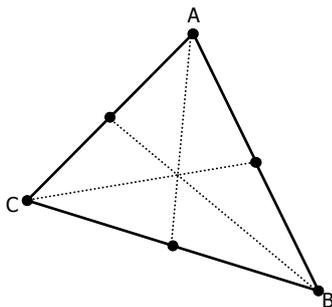**Example 10:** Find the perpendicular bisector of the line joining F (-4, 2) and G (6, 8).

> To find the equation of a perpendicular bisector:
>
> - Find the gradient of the line joining the given points
> - Find the perpendicular gradient (flip and make negative)
> - Find the coordinates of the midpoint
> - Substitute into y – b = m(x – a)

Lines Inside Triangles:
Medians, Altitudes & Perpendicular Bisectors

In a triangle, a line joining a corner to the **midpoint** of the opposite side is called a **median.**

A line through a corner which is **perpendicular** to the opposite side is called an **altitude.**

A line at 90° to the midpoint is called a **perpendicular bisector.**

The medians are **concurrent** (i.e. meet at the same point) at the **centroid**, which divides each median in the ratio 2:1
The median always divides the area of a triangle in half.
A solid triangle of uniform density will balance on the centroid.
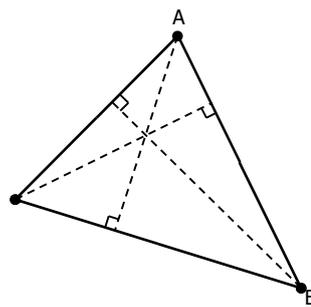
The altitudes are concurrent at the **orthocentre.** The orthocentre isn't always located inside the triangle e.g. if the triangle is obtuse.
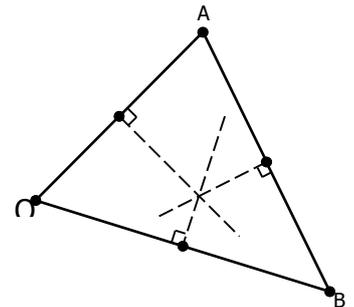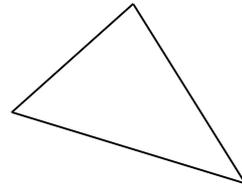
The perpendicular bisectors are concurrent at the **circumcentre.** The circumcentre is the centre of the circle touched by the vertices of the triangle.

For all triangles, the centroid, orthocentre and circumcentre are **collinear.**

Example 11: A triangle has vertices P (0, 2), Q (4, 4) and R (8, -6).

a) Find the equation of the median through P.

To find the equation of a median:

- Find the midpoint of the side opposite the given point
- Find the gradient of the line joining the given point and the midpoint
- Substitute into $y - b = m(x - a)$

b) Find the equation of the altitude through R.

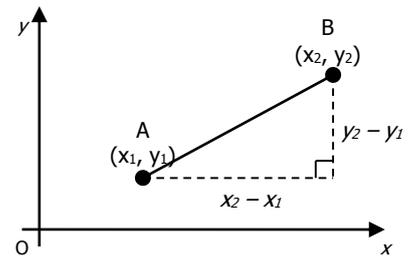To find the equation of an altitude:

- Find the gradient of the side opposite the given point
- Find the perpendicular gradient (flip and make negative)
- Substitute into $y - b = m(x - a)$

Distance between Two Points

The distance between any two points A (x_1, y_1) and B (x_2, y_2) can be found easily by Pythagoras' Theorem.

If d is the distance between A and B, then:

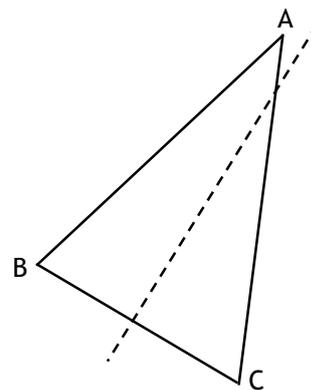$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 12: Calculate the distance between:

a) A (-4, 4) and B (2, -4)

b) X (11, 2) and Y (-2, -5)

Example 13: A is the point (2, -1), B is (5, -2) and C is (7, 4). Show that BC = 2AB.

Past Paper Example 1: The vertices of triangle ABC are A(7, 9), B(-3, -1) and C(5, -5) as shown:
The broken line represents the perpendicular bisector of BC

a) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$

b) Find the equation of the median from C

c) Find the co-ordinates of the point of intersection of the perpendicular bisector of BC and the median from C.

Past Paper Example 2:

The line GH makes an angle of 30° with the *y*-axis as shown in the diagram opposite.

What is the gradient of GH?