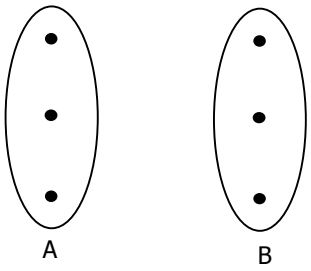
Inverse Functions

If a function links every number in the domain to **only** one number in the range, the function is called a **one to one correspondence**.

When function f(x) is a one to one correspondence from A to B, the function which maps from B back to A is called the **inverse function**, written $\mathbf{f^{-1}(x)}$.

For example, if f(x) = 2x, the inverse would be the function which "cancels out" multiplication by 2, i.e. $f^{-1}(x) = \frac{1}{2}x$



Finding the Formula of an Inverse Function

We can find the formula for the inverse of a function through a process very similar to changing the subject of a formula.

Example 1: For each function shown find a formula for the inverse function.

a) f(x) = 2x + 5

b) g(x) = $\frac{1}{2}(x - 9)$

c) p(x) = $3x^3 - 4$

d) $h(x) = \frac{3x + 17}{x - 4}$

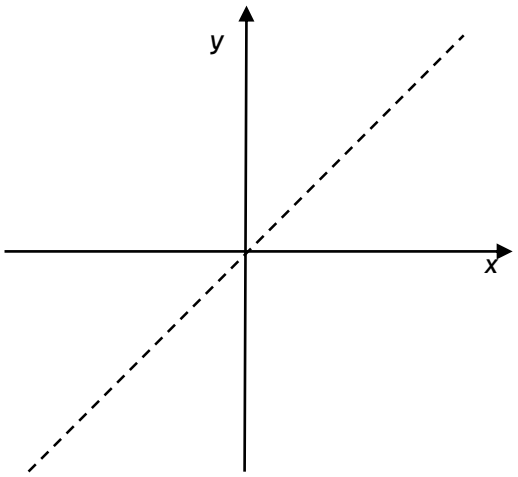
If f(g(x)) = x , then f(x) and g(x) are inverse functions, so that

f(x) = $g^{-1}(x)$ **AND** **g(x) = $f^{-1}(x)$**

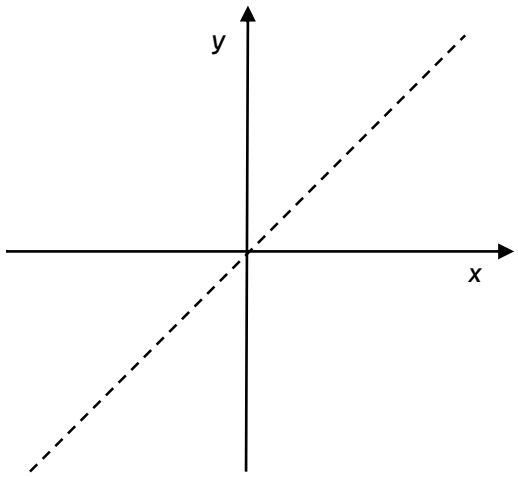
Example 2: f(x) = 2x + 5 and g(x) = $\frac{x-5}{2}$. Show that g(x) = $f^{-1}(x)$

Example 3: Sketch on the same graphs below:

a) $y = f(x)$ and $y = f^{-1}(x)$ where $f(x) = 2x + 5$



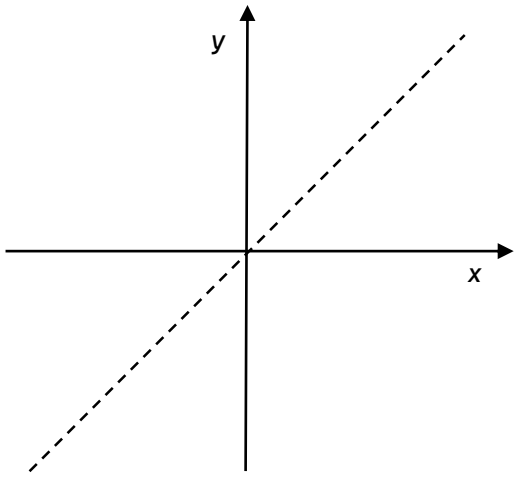
b) $y = f(x)$ and $y = f^{-1}(x)$ where $f(x) = \frac{x}{2} - 9$



The dotted lines on each diagram are the line y = x. In each case, the graph of an inverse function can be obtained from the graph of the original function by **reflecting in the line y = x.**

Example 4: $g(x) = x^3 + 6$

a) Sketch the graph of $y = g(x)$.



b) Show that $g^{-1}(x) = \sqrt[3]{x-6}$

c) Hence sketch the graph of $y = \sqrt[3]{x-6}$