

Finding Roots of Complex Numbers

Example: Find the cube roots of $1 + i$, leaving your answers in exponential form with $-\pi < \theta \leq \pi$

Step 1:

Find the modulus and argument of the complex number

$$1 + i \text{ has a modulus of } \sqrt{2} \text{ and an argument of } \frac{\pi}{4}$$

Step 2:

Write the complex number in exponential form

$$1 + i = \sqrt{2}e^{i\frac{\pi}{4}}$$

Step 3:

Write your equation equal to the exponential form of the complex number, but when you write the argument put $+2\pi k$ to make sure you get all the solutions

$$z^3 = \sqrt{2}e^{i\left(\frac{\pi}{4} + 2\pi k\right)}$$

Step 4:

Find the root of both sides of this equation

$$z = 2^{\frac{1}{6}}e^{i\left(\frac{\pi}{12} + \frac{2\pi k}{3}\right)}$$

Step 5:

Put in the values of k that give you the correct number of solutions, and angles in the range you want

Since this is a cube root, we want 3 values of k . For $-\pi < \theta \leq \pi$ we want $k = -1, 0, 1$ (Had our range been $0 \leq \theta < 2\pi$ we would need $k = 0, 1, 2$)

$$\text{Solutions are } z = 2^{\frac{1}{6}}e^{-\frac{7\pi}{12}}, 2^{\frac{1}{6}}e^{\frac{\pi}{12}}, 2^{\frac{1}{6}}e^{\frac{3\pi}{4}}$$