Loci in the Complex Plane Questions

1. (a) Draw an Argand diagram to show the points A and B which represent the complex numbers $1 - 3i$ and $5 - i$ respectively.

  (b) (i) The circle C has AB as a diameter. Find its radius and the coordinates of its centre.

     (ii) Write down the equation of C in the form $|z - z_0| = k$.

2. The complex numbers z_1 and z_2 are given by $z_1 = 1 + \sqrt{3}\, i$ and $z_2 = iz_1$

  (a) (i) Express z_2 in the form $a + ib$.

     (ii) Find the modulus and argument of z_2.

  (b) Label the points representing z_1 and z_2 on an Argand diagram.

  (c) On the **same** Argand diagram, sketch the locus of points z satisfying:

     (i) $|z - z_1| = |z - z_2|$

     (ii) $\arg(z - z_1) = \arg z_2$

3. The point P in the complex plane represents the complex number $z = x + i\,y$. **Describe** the locus of P in each of the following cases:

  (a) $|z - 4 + 3i| = 3$

  (b) $|z - 4 + 3i| = |z - i|$

4. (a) Shade, on an Argand diagram, the region in which $|z - 2i| \le 1$

  (b) Find the greatest and least values of the argument of complex numbers z satisfying $|z - 2i| \le 1$ giving your answers in terms of π.

5. The complex numbers u, v and w are given by $u = 21 + 12i$, $v = 12 + 0i$ and $w = 0 + 21i$, and are represented in the complex plane by the points U, V and W respectively. The variable point P in the plane represents the complex number $z = x + iy$.

  (a) On **separate** sketches of the complex plane, draw the locus of P in each of the following cases:

     (i) $|z - u| = 12$

     (ii) $\arg(z - v) = \dfrac{\pi}{4}$

     (iii) $|z - v| = |z + w|$

  (b) In the situation when $|z - u| = 2|z - v|$, the locus of P is a circle. Find the radius of this circle and the complex number corresponding to its centre.

Brief Solutions to Loci Questions

1. (b) (i) The centre is 3 - 2i . The radius is $\sqrt{5}$

    (ii) The equation is $|z - 3 + 2i| = \sqrt{5}$

2. (a) (i) $-\sqrt{3} + i$

    (ii) $|z_2| = 2$, arg $z_2 = \dfrac{5\pi}{6}$

    (c) (i) Perpendicular bisector
        Through (0, 0)
    (ii) Half line through z_1
        Parallel to Oz_2

3. (a) Circle: Centre 4 – 3i and radius 3

    (b) Perpendicular bisector (e.g. straight line) of line segment joining 4 – 3i to i

4. (b) least argument $\dfrac{\pi}{3}$ greatest argument $\dfrac{2\pi}{3}$

5. (a)(i) Circle, centre (21,12) radius 12

    (ii) Half line at $\pi/4$ to real-axis, starting from (12, 0)

    (iii) Perpendicular bisector of line segment joining $v = 12 + 0i$ and $-w = 0 – 21i$

    (b) circle has centre at 9 – 4 i and radius 10