Loci in the Complex Plane

A locus is a set of points, z, satisfying an equation (or inequality), usually shown as a curve or line.

1. $|z-a|=r$

This is a circle centre a, with radius r

2. $\arg(z-a)=\alpha$

This is a half line, starting at a, at an angle of α to the positive *x* axis (the real axis)

3. $|z-z_1|=|z-z_2|$

This is the perpendicular bisector of the line joining z_1 to z_2

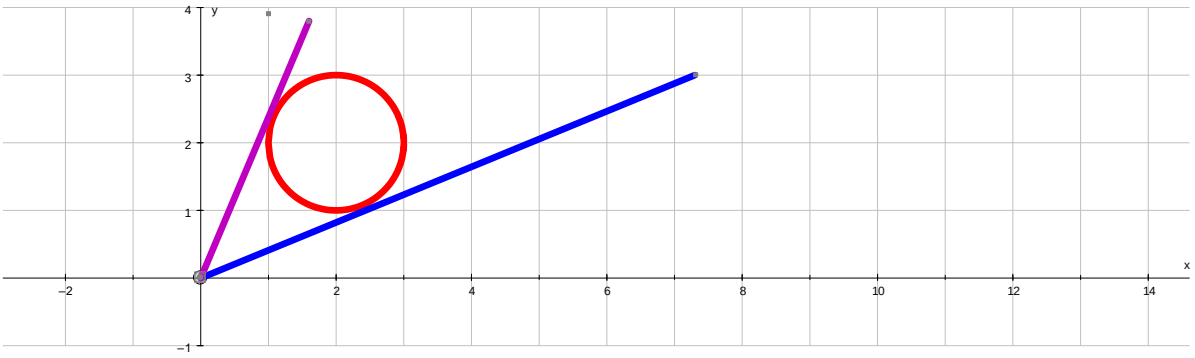
4. $|z-z_1|=k|z-z_2|$

When k is not equal to 1, this is a circle, but we need to substitute in $z=x+iy$ and expand, to find out the equation of the circle

5. $\arg\left(\frac{z-z_1}{z-z_2}\right)=\alpha$

This is an arc of the circle which passes through z_1 and z_2, going anticlockwise from z_1 to z_2, such that if A represents z_1, B represents z_2, and P is a point on the locus, then angle APB is α

If the locus of z is a circle, then the maximum and minimum values of $\arg(z)$ occur where the tangents through the origin touch the circle



If the locus of z is a circle, then the maximum and minimum values of $|z|$ are the points on the circle on the line joining the origin to the centre of the circle