St. Andrew's and St. Bride's High School

Advanced Higher Homework 8

1. Express in the form of $x + yi$:

   (a) $\dfrac{5 - i}{i}$ (b) $\dfrac{2}{3 + i}$ (c) $\dfrac{3 + 4i}{2 - 5i}$ (d) $\dfrac{\cos A - i \sin A}{\cos A + i \sin A}$

2. Find: $\sqrt{-8 + 6i}$

3. Find the modulus and argument of: (a) $-1 + \sqrt{3}\, i$ (b) $-1 - i$

4. If $z_1 = 8(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$ and $z_2 = 2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$, find (in polar form):

   (a) $z_1 z_2$ (b) $\dfrac{z_1}{z_2}$

5. If $z = \cos\theta + i \sin\theta$, find $z^3 + \dfrac{1}{z^3}$.

6. Write $(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12})^8$ in the form $x + yi$ (**not** in polar form).

7. By considering $(\cos\theta + i \sin\theta)^4$ and using De Moivre's Theorem, deduce a trigonometric identity for $\cos 4\theta$ in terms of $\cos\theta$.

8. Given that $2 - 3i$ is a root of the equation $z^4 - 6z^3 + 26z^3 - 46z + 65 = 0$, find all of the other roots.

9. Find the set of points, z, where $|z - (2 - 5i)| = 6$, explaining its geometrical meaning.