St. Andrew's and St. Bride's High School

Advanced Higher Homework 6

1. By considering the expansion of $(\cos\theta + i\sin\theta)^4$, express:

   (i) cos 4θ in terms of cosθ
   (ii) sin 4θ in terms of sinθ and cosθ
   (iii) $\cos^4\theta$ in terms of cosθ and cos 4θ

2. $1 + 2i$ is a root of the equation $z^4 - 5z^3 + 13z^2 - 19z + 10$. Find the real factors of the polynomial.

3. In the following :

   (i) show that the given complex number is a zero of the given polynomial,
   (ii) find all the remaining roots

   $z = 2 + i$; $f(z) = z^4 - 2z^3 - z^2 + 2z + 10$

4. Given that $z = x + yi$, for each of the following, find the equation of the locus.

   (a) $|z - 3| = 2$ (b) $|z + 1 + 2i| = 3$ (c) $|z - 2| = |z - i|$