Advanced Higher Maths
SQA 2025 Paper 2
Question 18

Let $z = x + iy$ be a complex number, where $x, y \in \mathbb{R}$.

(a) (i) Express $\overline{z} + iz$ in Cartesian form, where $\overline{z}$ is the complex conjugate of z. **2**

(ii) Given $x > y$, find the argument of $\overline{z} + iz$. **1**

When $x < y$, $\overline{z} + iz = r\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$ where r is the modulus of $\overline{z} + iz$.

(b) Use de Moivre's theorem to find, in polar form, both square roots of $\overline{z} + iz$. **2**

Answers:

(a) (i) $x - y + i(x - y)$

(ii) $\frac{\pi}{4}$ or $45°$

(b) $\sqrt{r}\left(\cos\left(-\frac{3\pi}{8}\right) + i\sin\left(-\frac{3\pi}{8}\right)\right)$

$\sqrt{r}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$