Advanced Higher Maths
SQA 2022 Paper 2
Question 12

Let $z = \cos\theta + i\sin\theta$.

(a) Use de Moivre's theorem to state an expression for z^4. **1**

(b) State and simplify the binomial expansion of $(\cos\theta + i\sin\theta)^4$. **3**

(c) Hence show that:

(i) $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$. **2**

(ii) $\sin\theta\cot 4\theta$ can be written in terms of $\cos\theta$ only. **2**

Answers:

(a) $\cos 4\theta + i\sin 4\theta$

(b) $\cos^4\theta + 4\cos^3\theta i\sin\theta - 6\cos^2\theta\sin^2\theta$
$-4\cos\theta i\sin^3\theta + \sin^4\theta$

(c) (i) Equate real parts and express in terms of only $\cos\theta$.

(ii) $\dfrac{8\cos^4\theta - 8\cos^2\theta + 1}{8\cos^3\theta - 4\cos\theta}$