Advanced Higher Maths
SQA 2021 Paper 2
Question 13

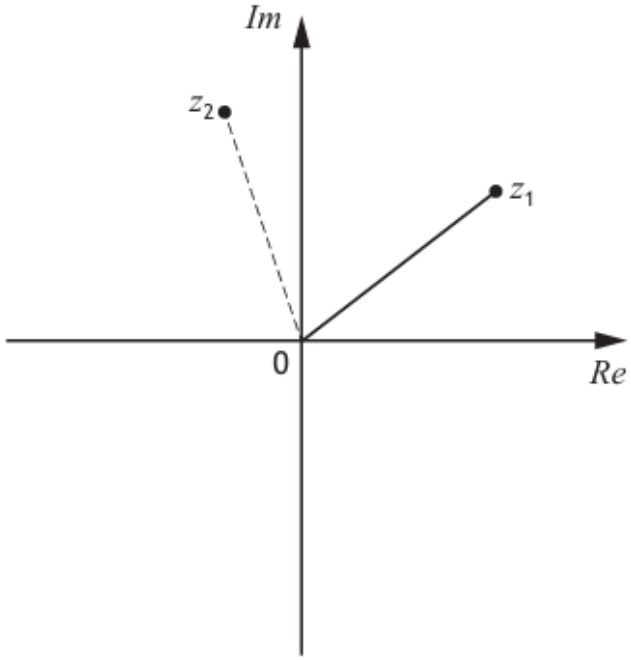
Advanced Higher Maths
Maths.scot

(a) Express -1 in the form $\cos\theta + i\sin\theta$. 1

The complex number z_1 is defined by $z_1 = \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$.

(b) Use de Moivre's theorem to show that z_1 is a root of the equation $z^5 + 1 = 0$. 1

The complex number z_2 is also a root of the equation $z^5 + 1 = 0$. Roots z_1 and z_2 have been plotted on an Argand diagram, as shown.



(c) Express z_2 in the form $\cos\theta + i\sin\theta$. 1

The remaining roots of the equation $z^5 + 1 = 0$ are z_3, z_4 and z_5.

(d) Express z_3, z_4 and z_5 in the form $\cos\theta + i\sin\theta$, where $-\pi < \theta \leq \pi$. 2

(e) Given $z_1 + z_2 + z_3 + z_4 + z_5 = 0$, show algebraically that

$$\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} = \frac{1}{2}.$$ 2

Answers:

(a) $-1 = \cos\pi + i\sin$

(b) $\cos\pi + \mathrm{i}\sin\pi$

(c) $z_2 = \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$

(d) $\cos\left(-\frac{\pi}{5}\right) + \mathrm{i}\sin\left(-\frac{\pi}{5}\right)$ or

$\cos\left(-\frac{3\pi}{5}\right) + \mathrm{i}\sin\left(-\frac{3\pi}{5}\right)$ or

$\cos\pi + i\sin\pi$

(e) $\cos\frac{\pi}{5} + \cos\frac{3\pi}{5} = \frac{1}{2}$