

**Advanced Higher Maths**  
**SQA 2016 Paper**  
**Question 10**



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For each of the following statements, decide whether it is true or false.

If true, give a proof; if false, give a counterexample.

- A. If a positive integer  $p$  is prime, then so is  $2p + 1$ .
- B. If a positive integer  $n$  has remainder 1 when divided by 3, then  $n^3$  also has remainder 1 when divided by 3.

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Answer:

A is false. One counterexample is  $p = 7$ , because  $2 \times 7 + 1 = 15$ , which is not prime.

B is true. Direct proof, eg.  $n = 3a + 1 \Rightarrow n^3 = 3(9a^3 + 9a^2 + 3a) + 1$ , hence remainder 1.