(a) Given $z = \cos\theta + i\sin\theta$, use de Moivre's theorem and the binomial theorem to show that:

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$$

and

$$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta.$$ **5**

(b) Hence show that $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$.   **3**

(c) Find algebraically the solutions to the equation

$$\tan^4\theta + 4\tan^3\theta - 6\tan^2\theta - 4\tan\theta + 1 = 0$$

in the interval $0 \le \theta \le \dfrac{\pi}{2}$.   **3**

Answers:

(a)   Proof. See marking instructions.

(b)   Proof. See marking instructions.

(c) $\theta = \dfrac{\pi}{16}$ and $\dfrac{5\pi}{16}$