Prelim Revision **Complex Numbers**

1) Write each of the following in the form $a + bi$.

a) $(4 + 3i) + (3 - i)$ b) $2i\,(3 + 3i)$ c) $\dfrac{4+5i}{1+7i}$ d) $\dfrac{2+4i}{9+i}+\dfrac{2+i}{5-2i}$

2) For each of the following, plot z on an Argand Diagram, calculate the modulus and argument and write z in polar form.

a) $z = 2 + 4i$ b) $z = 2 - 4i$ c) $z = -1 + 7i$ d) $-5 - 8i$

3) For 2a) above. Use De Moivres Theorem to calculate the following giving your answer in the form $a + bi$, correct to 1 d.p.

a) z^2 b) z^6 c) z^{10} d) $\sqrt{z}$

e) $\sqrt[3]{z}$ f) $\sqrt{z^3}$

4) Solve $z^3 + z^2 - 7z - 15 = 0$ given that $z - 3$ is a factor.

5) a) Show that $z = 2 + i$ is a factor of $z = z^4 - 2z^3 - z^2 + 2z + 10$.

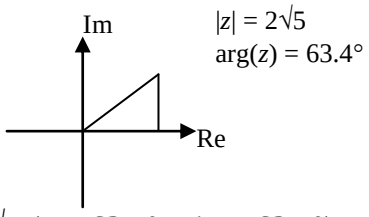
b) Write down another solution of the polynomial

c) Find the other two roots.

Answers

1a) $7 + 2i$ b) $-6 + 6i$ c) $\frac{39}{50} - \frac{23}{50}i$ d) $\frac{647}{1189} + \frac{920}{1189}i$

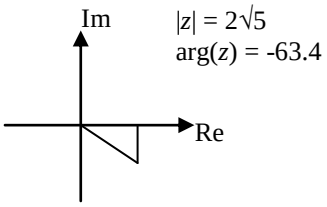
2a)



$|z| = 2\sqrt{5}$
$\arg(z) = 63.4°$

$z = 2\sqrt{5}\ (\cos 63.4° + i \sin 63.4°)$

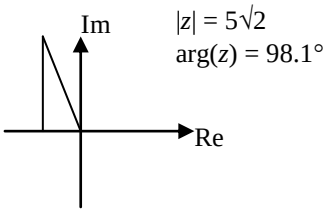
b)



$|z| = 2\sqrt{5}$
$\arg(z) = -63.4$

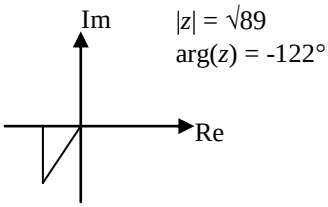
$z = 2\sqrt{5}\ (\cos(-63.4) + i \sin(-63.4))$

c)



$|z| = 5\sqrt{2}$
$\arg(z) = 98.1°$

$z = 5\sqrt{2}\ (\cos 98.1 + i \sin 98.1°)$



$|z| = \sqrt{89}$
$\arg(z) = -122°$

$z = \sqrt{89}\ (\cos(-122°) + i \sin(-122°))$

3a) $-12 + 16i$ b) $7489.3 + 27688.6i$ c) $223220.7 - 319206i$

d) $1.8 + 1.1i$ e) $1.5 + 0.6i$ f) $-0.8 + 9.4i$

4) $3, -2 \pm i$

5)a) proof b) $2 - i$ c) $2 \pm i, -1 \pm i$