AH # Topic PP – Summation & Induction *Unit 2 Outcome 2.3*

2001

A4. Prove by induction that for all integers n ≥ 1, $2 + 5 + 8 + \ldots + (3n - 1) = \frac{1}{2}n(3n + 1)$. 5 marks

2002

A7. Prove by induction that $4^n - 1$ is divisible by 3 for all positive integers n. 5 marks

2003

A2. Given that $u_k = 11 - 2k,\ (k \geq 1)$, obtain a formula for $S_n = \sum_{k=1}^{n} u_k$. 3 marks

Find the values of n for which $S_n = 21$. 2 marks

B5. a) Prove by induction that for all natural numbers n ≥ 1 $\sum_{r=1}^{n} 3(r^2 - r) = (n - 1)n(n + 1)$. 4 marks

2004

12. Prove by induction that $\frac{d''}{dx''}(xe^x) = (x + n)e^x$ for all integers $n \geq 1$. 5 marks

2005

10. Prove by induction that, for all positive integers n, $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$. 5 marks

2007

12. Prove by induction that for a > 0, $(1 + a)^n \geq 1 + na$ for all positive integers n. 5 marks

2008

9. Show that $\sum_{r=1}^{n}(4 - 6r) = n - 3n^2$. 2 marks

Hence write down a formula for $\sum_{r=1}^{2q}(4 - 6r)$. 1 mark

Show that $\sum_{r=q+1}^{2q}(4 - 6r) = q - 9q^2$. 2 marks

2009

4. Prove by induction that, for all positive integers n, $\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$. 5 marks

12. The first two terms of a geometric sequence are $a_1 = p$ and $a_2 = p^2$.

Obtain expressions for S_n and S_{2n}, in terms of p, where $S_k = \sum_{j=1}^{k} a_j$. 1, 1 marks

Given that $S_{2n} = 65S_n$ show that $p^n = 64$. 2 marks

Given also that $a_3 = 2p$ and that $p > 0$, obtain the exact value of p and hence the value of n.

1, 1 marks

2011

8. Write down an expression for $\sum_{r=1}^{n} r^3 - \left(\sum_{r=1}^{n} r\right)^2$ and an expression for $\sum_{r=1}^{n} r^3 + \left(\sum_{r=1}^{n} r\right)^2$. 1 mark

3 marks

12. Prove by induction that $8^n + 3^{n-2}$ is divisible by 5 **for all integers $n \geq 2$.** 5 marks

2012

16. (a) Prove by induction that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

for all integers $n \geq 1$. 6 marks

(b) Show that the real part of $\dfrac{\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)^{11}}{\left(\cos\frac{\pi}{36} + i\sin\frac{\pi}{36}\right)^{4}}$ is zero. 4 marks

2013

9. Prove by induction that, for all positive integers n, $\sum_{r=1}^{n} (4r^3 + 3r^2 + r) = n(n+1)^3$. 4 marks

2016

5. Prove by induction that $\sum_{r=1}^{n} r(3r-1) = n^2(n+1), \quad \forall n \in \bullet$. 4 marks

2018

12. Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} 3^{r-1} = \frac{1}{2}(3^n - 1).$$ 5

2019

14. Prove by induction that

$\sum_{r=1}^{n} r!r = (n+1)! - 1$ for all positive integers n. **5**

Answers

2003 A2 a) $S_n = -n^2 + 10n$ or $S_n = \frac{n}{2}[18 + (n-1)(-2)]$ **(3)** b) $n = 3$ and 7. **(2)**

2007 9.a) Proof b) $2q - 12q^2$ c) Proof

2009 12.a) $S_n = \frac{p(p^n - 1)}{p-1}$; $S_{2n} = \frac{p(p^{2n} - 1)}{p-1}$ **(2)** b) Proof **(2)** c) $p = \sqrt{2}$ $n = 12$ **(2)**

2011 8. $\sum_{k=1}^{n} r^3 = \left[\sum_{k=1}^{n} r\right]^2$ $\frac{1}{2}n^2(n+1)^2$

2018 12. Proof **2019** Proof