2001

A8. A function f is defined by $f(x)=\dfrac{x^2+6x+12}{x+2}, x\neq -2.$

(a) Express $f(x)$ in the form $ax+b+\dfrac{b}{x+2}$ stating the values of a and b. 2 marks

(b) Write down an equation for each of the two asymptotes. 2 marks

(c) Show that $f(x)$ has two stationary points. 4 marks
Determine the coordinates and the nature of the stationary points.

(d) Sketch the graph of f. 1 mark

(e) State the range of values of k such that the equation $f(x) = k$ has no solution. 1 mark

2002

A8. Express $\dfrac{x^2}{(x+1)^2}$ in the form $A+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}, (x\neq -1),$ stating the values of the constants A, B and C.

A curve is defined by $y=\dfrac{x^2}{(x+1)^2}, (x\neq -1).$ 3 marks

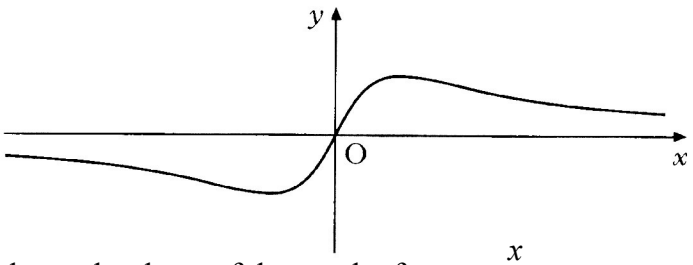
(i) Write down equations for its asymptotes. 2 marks

(ii) Find the stationary point and justify its nature. 4 marks

(iii) Sketch the curve showing clearly the features found in (i) and (ii). 2 marks

2003

A7.



The diagram shows the shape of the graph of $y=\dfrac{x}{1+x^2}$.

Obtain the stationary points of the graph. 4 marks

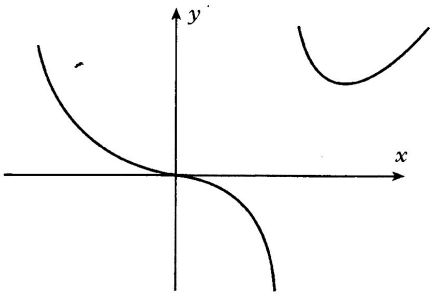
Sketch the graph of $y=\left|\dfrac{x}{1+x^2}\right|$ and identify its three critical points. 3 marks

2004

10. Determine whether the function $f(x)=x^4\sin 2x$ is odd, even or neither. 3 marks
Justify your answer.

2005

11. The diagram shows part of the graph of $y = \frac{x^3}{x-2}, x \neq 2.$



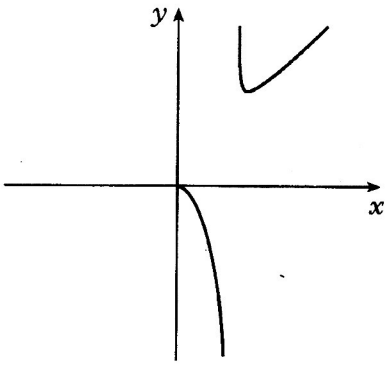
a) Write down the equation of the vertical asymptotes. 1 mark

b) Find the coordinates of the stationary points of the graph of $y = \frac{x^3}{x-2}$. 4 marks

c) Write down the coordinates of the stationary points of the graph of $y = \left|\frac{x^3}{x-2}\right| + 1.$ 2 marks

2006

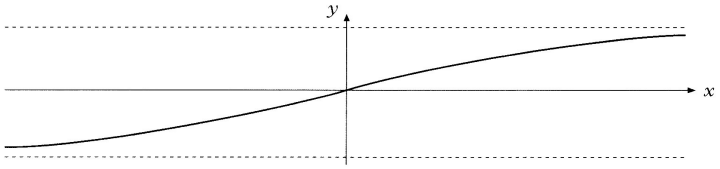
12. The diagram shows part of the graph of a function f which satisfies the following conditions:
(i) f is an even function;
(ii) two of the asymptotes of the graph y = f(x) are y = x and x = 1.
Copy the diagram and complete the graph. Write down equations for the other two asymptotes.



3 marks

2007

16.



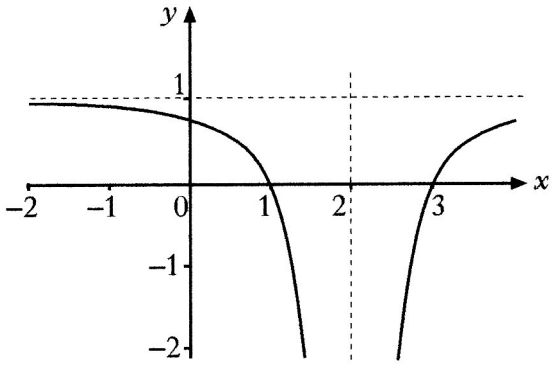
(a) The diagram shows part of the graph of $f(x) = \tan^{-1}2x$ and its asymptotes. State the equations of these asymptotes. 2 marks

(b) Use integration by parts to find the area between $f(x)$, the x-axis and the lines $x = 0, x = \frac{1}{2}$. 5 marks

(c) Sketch the graph of $y = |f(x)|$ and calculate the area between this graph, the x-axis and the lines $x = -\frac{1}{2}, x = \frac{1}{2}$. 3 marks

2008

3. Part of the graph $y = f(x)$ is shown below, where the dotted lines indicate asymptotes. Sketch the graph of $y = -f(x + 1)$ showing its asymptotes. Write down the equations of the asymptotes. 4 marks



2009

1, 1 marks

13. A function $f(x)$ is defined by $f(x) = \dfrac{x^2 + 2x}{x^2 - 1} \qquad (x \neq \pm 1)$.

Obtain equations for the asymptotes of the graph of $f(x)$. 3 marks

Show that $f(x)$ is strictly decreasing function. 3 marks

Find the coordinates of the points where the graph of $f(x)$ crosses

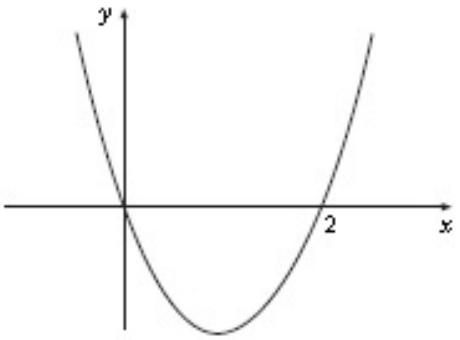
(i) the x-axis and

(ii) the horizontal asymptote. 2 marks

Sketch the graph of $f(x)$, showing clearly all relevant features. 2 marks

2010

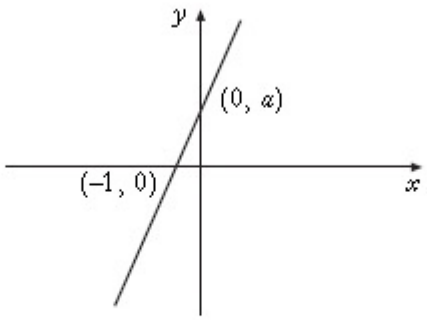
10. The diagram below shows part of the graph of a function $f(x)$. State whether $f(x)$ is odd, even or neither. Fully justify your answer.



3 marks

2011

6.



The diagram shows part of the graph of a function $f(x)$. Sketch the graph of $|f^{-1}(x)|$ showing the points of intersection with the axes. 4 marks

2012

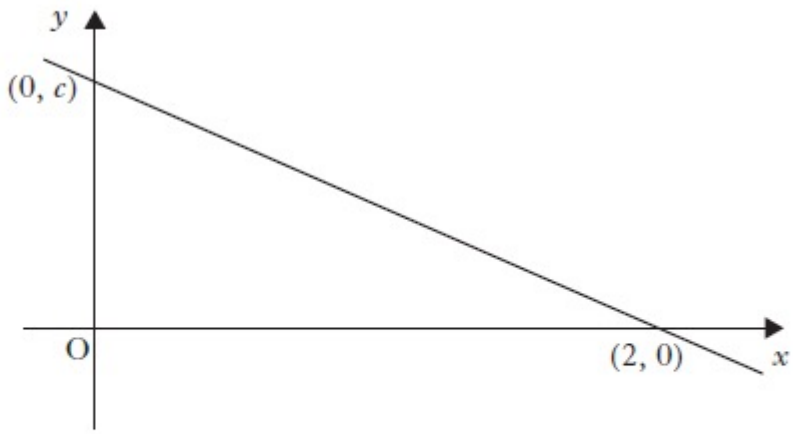
7. A function is defined by $f(x) = |x + 2|$ for all x.

(*a*) Sketch the graph of the function for $-3 \leq x \leq 3$. 2 marks

(*b*) On a separate diagram, sketch the graph of $f'(x)$. 2 marks

2013

13. Part of the straight line graph of a function $f(x)$ is shown.



(*a*) Sketch the graph of $f^{-1}(x)$, showing points of intersection with the axes. 2 marks

(*b*) State the value of k for which $f(x) + k$ is an odd function. 1 mark

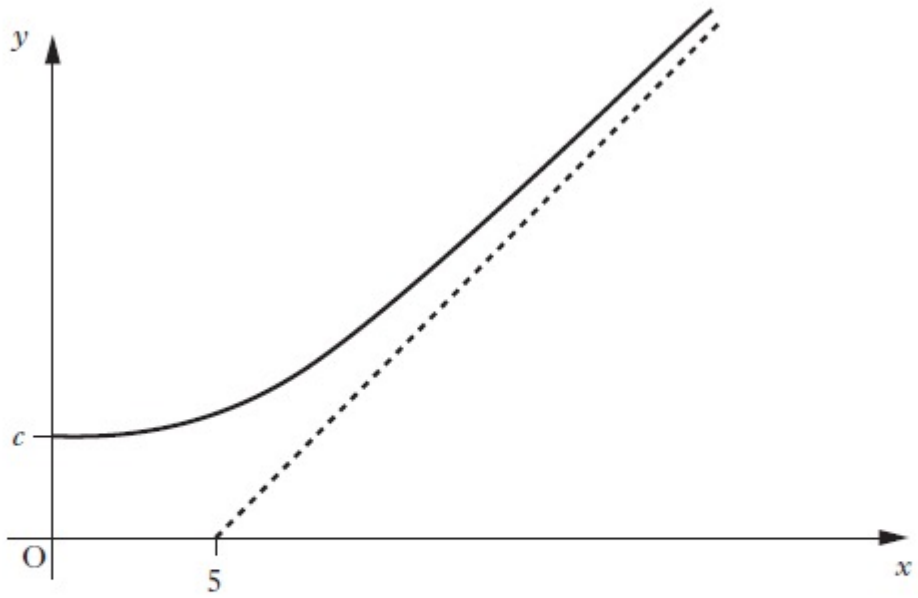
(*c*) Find the value of h for which $|f(x + h)|$ is an even function. 2 marks

2014

11. The function $f(x)$ is defined for all $x \geq 0$.

The graph of $y = f(x)$ intersects the y-axis at $(0, c)$, where $0 < c < 5$.

The graph of the function and its asymptote, $y = x - 5$, are shown below.



(*a*) Copy the above diagram.

On the same diagram, sketch the graph of $y = f^{-1}(x)$.

Clearly show any points of intersection and any asymptotes. 4 marks

(*b*) What is the equation of the asymptote of the graph of $y = f(x + 2)$? 1 mark

(*c*) Why does your diagram show that the equation $x = f(f(x))$ has at least one solution? 1 mark

2015

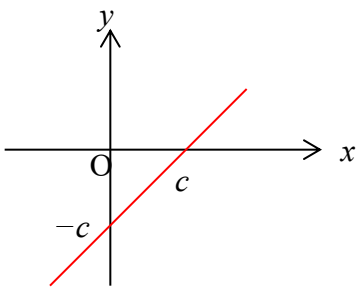
14. For some function, f, define

$$g(x) = f(x) + f(-x) \quad \text{and}$$
$$h(x) = f(x) - f(-x).$$

Show that $g(x)$ is an even function and that $h(x)$ is an odd function.
Hence show that $f(x)$ can be expressed as the sum of an even and an odd function. 4 marks

2016

12. Below is a diagram showing the graph of a linear function $y = f(x)$.



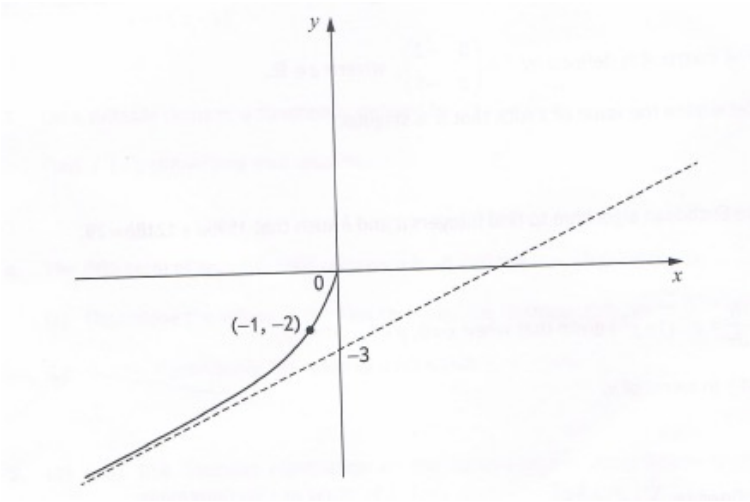
2,2 marks

On separate diagrams show: a) $y = |f(x) - c|$ b) $y = |2f(x)|$

2017

12. In the diagram below part of the graph of $y = f(x)$ has been omitted.

The point $(-1, -2)$ lies on the graph and the line $y = \frac{1}{2}x - 3$ is an asymptote.



Given that $f(x)$ is an odd function:

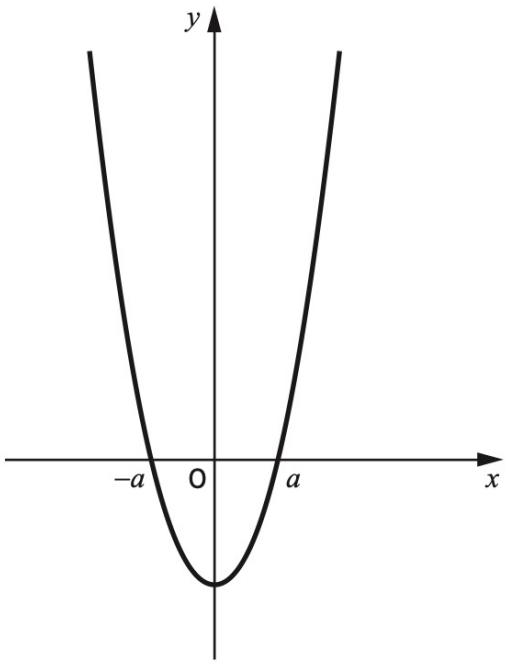
a) Copy and complete the diagram, including any asymptotes and any points you know to be on the graph. **2 marks**

b) $g(x) = |f(x)|$. On a separate diagram, sketch $g(x)$. Include known asymptotes and points. **2 marks**

c) State the range of values for $f'(x)$ given that $f'(0) = 2$. **1 mark**

2019

3. The function $f(x)$ is defined by $f(x) = x^2 - a^2$. The graph of $y = f(x)$ is shown in the diagram.



(a) State whether $f(x)$ is odd, even or neither. Give a reason for your answer. **1**

(b) Sketch the graph of $y = |f(x)|$. **1**

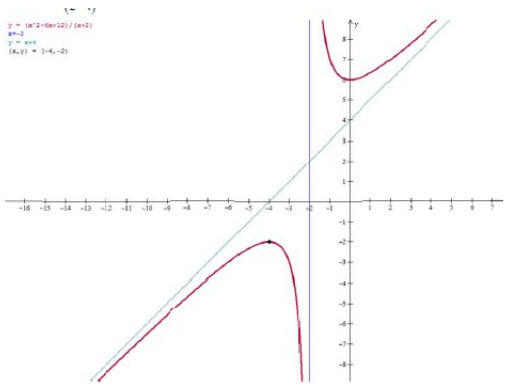
Answers

2001

A8. a) $a = 1,\ b = 4$ b) $x = -2,\ y = x + 4$

c) Max (-4, -2) Min (0, 6)

d) $-2 < k < 6$

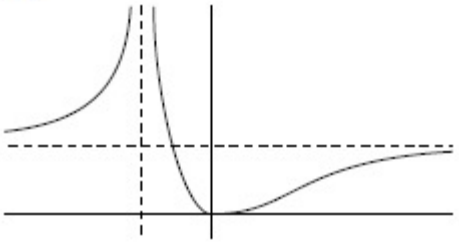


2002

A8. A = 1, B = -2, C = 1 a) $y = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$ $y = 1, x = -1$

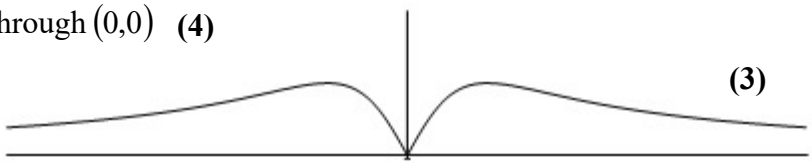
b) Min (0, 0)

(c)



2003

SP $= \left(1, \frac{1}{2}\right)$ and $\left(-1, -\frac{1}{2}\right)$ and passes through $(0,0)$ **(4)**



(3)

2004

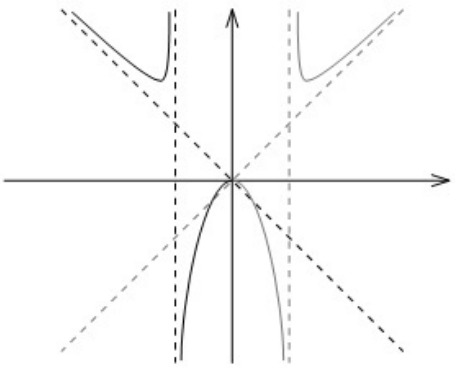
$f(-x) = -f(x)$ therefore is ODD. **(3)**

2005

11. a) $x = 2$ **(1)** b) (0, 0) and (3, 27) **(4)** c) (0, 1) and (3, 28) **(2)**

2006

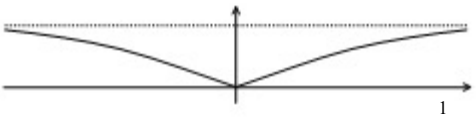
12.



$y = -x;\ x = -1$ **(3)**

2007

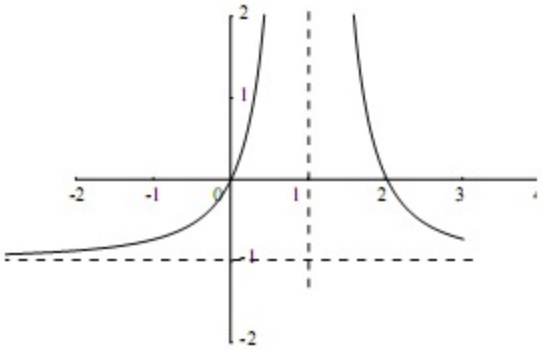
$y = \pm\frac{\pi}{2}$ **(2)** $\frac{\pi}{8} - \frac{1}{4}\ln 2$ **(5)**



$\int_{-\frac{1}{2}}^{\frac{1}{2}} |f(x)|dx = \frac{\pi}{4} - \frac{1}{2}\ln 2$ **(3)**

2008

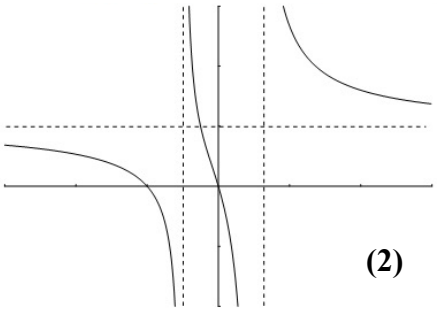
$f'(x) = \frac{-3}{\sqrt{1-9x^2}}$ **(2)** $f'(x) = \frac{3\cos^3\theta}{2\sin\theta}$ **(3)**



2009

$x = -1 \quad x = 1 \quad y = 1$ **(3)** $f'(x) < 0$ therefore always neg grad **(3)**

$x = 0 \quad x = -2$ Horizontal Asy $x = -\frac{1}{2}$ **(2)**



(2)

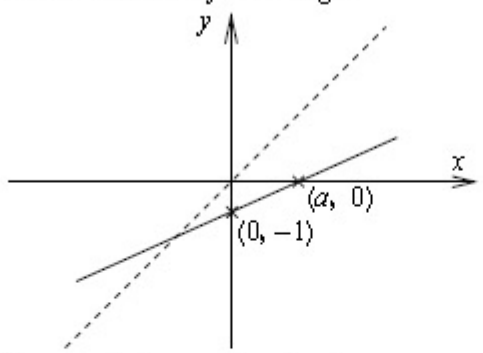
2010

10. The graph is not symmetrical about y – axis therefore it is NOT an even function.
The graph does not have half turn rotational symmetry therefore it is NOT an odd function.
The function is neither odd nor even.

2011

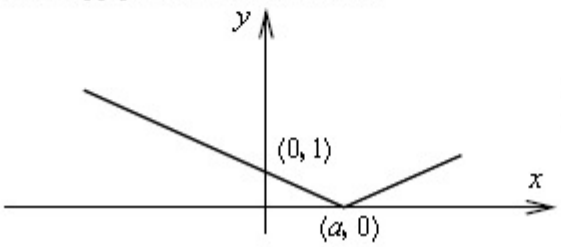
6. **(4)**

Reflect in the line $y = x$ to get



1 for position
1 for coordinates

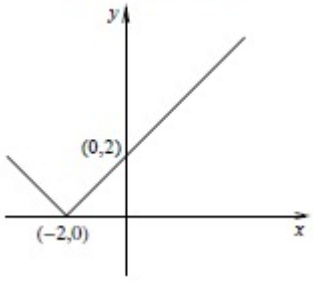
Now apply the modulus function



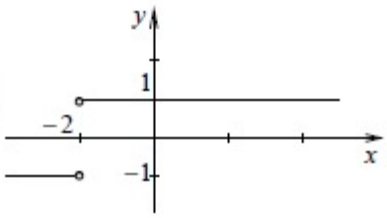
1 for shape
1 for coordinates

2012

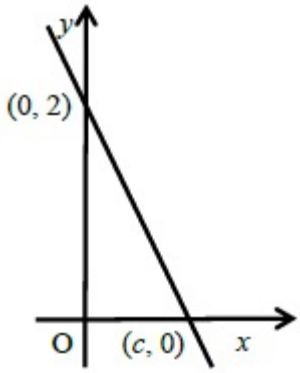
$y = |x + 2|$



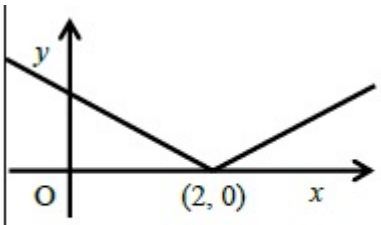
(b)



2013



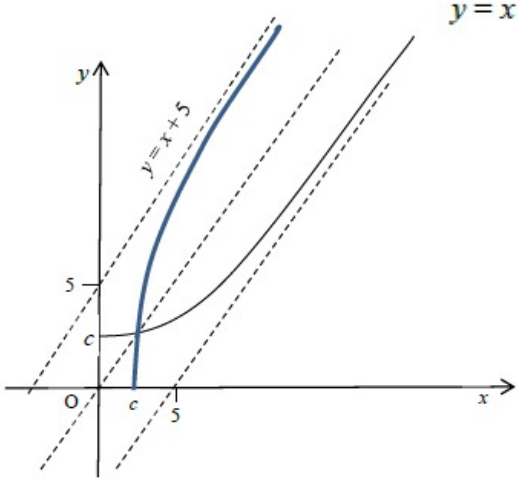
$y = f(x) - c$ is odd. $\therefore k = -c$



$y = |f(x+2)|$ is even $\therefore h = 2$

2014

$y = x - 3$



From the diagram, the two **curves/graphs** intersect

OR

$y = f(x)$ intersects $y = x$

OR

$y = f^{-1}(x)$ intersects $y = x$

OR

$f^{-1}(x) = f(x)$

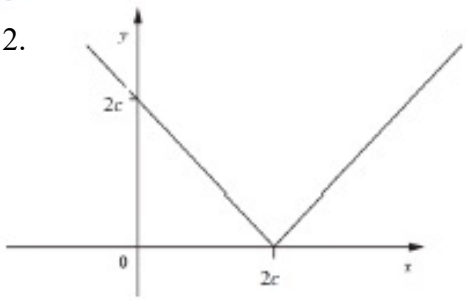
So $x = f(f(x))$

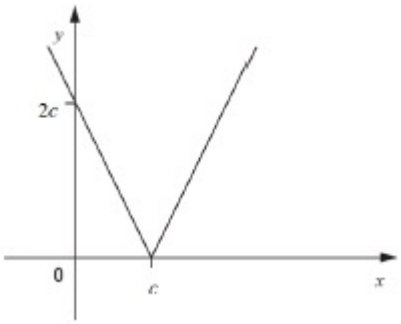
2015

Proof

2016

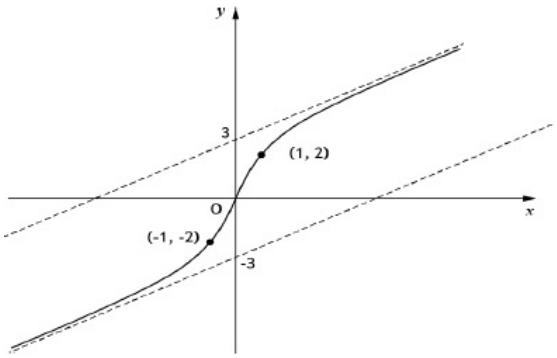
12.

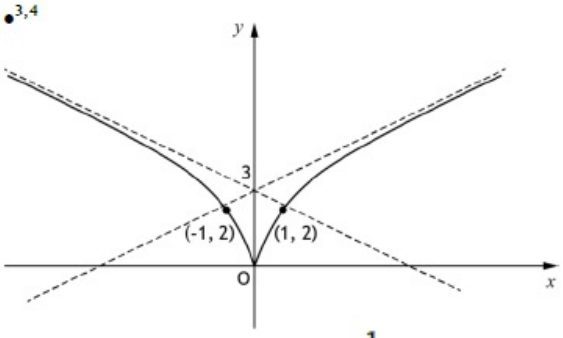




2017

12.





(5)

$\bullet^5\ \frac{1}{2} < f'(x) \leq 2$

2019

3. a) graph is symmetrical about the y-axis $\therefore$ even

OR

$f(-x) = (-x)^2 - a^2 = x^2 - a^2 = f(x) \therefore$ even

b)