2006

7. For all natural numbers n, prove whether the following results are true or false.

a) $n^3 - n$ is always divisible by 6.

b) $n^3 + n + 5$ is always prime. 5 marks

2008

11. For each of the following statements, decide whether it is true or false and prove your conclusion.

A For all natural numbers m, if m^2 is divisible by 4 then m is divisible by 4.
B The cube of any odd integer p plus the square of any even integer q is always odd. 5 marks

2010

8. (a) Prove that the product of two odd integers is odd. 2 marks

(b) Let p be an odd integer. Use the results of (a) to prove by induction that p^n is odd for all positive integers n. 4 marks

12. Prove by contradiction that if x is an irrational number, then $2 + x$ is irrational. 4 marks

2012

16. (a) Prove by induction that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ 6 marks

for all integers $n \geq 1$.

(b) Show that the real part of $\dfrac{\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)^{11}}{\left(\cos\frac{\pi}{36} + i\sin\frac{\pi}{36}\right)^{4}}$ is zero. 4 marks

2015

9. Show that $\binom{n+2}{3} - \binom{n}{3} = n^2$, for all integers, n, where $n \geq 3$. 4 marks

12. Prove that the difference between the squares of any two consecutive odd numbers is divisible by 8. 3 marks

2016

10. For each of the following statements, decide whether it is true or false.
If true, give a proof; if false, give a counterexample.

A. If a positive integer p is prime, then so is $2p + 1$.
B. If a positive integer n has remainder 1 when divided by 3, then n^3 also has remainder 1 when divided by 3. 4 marks

2017

13. Let n be an integer.

Using proof by contrapositive, show that if n^2 is even, then n is even. 4 marks

2018

9. Prove directly that:

(a) the sum of any three consecutive integers is divisible by 3; 2

(b) any odd integer can be expressed as the sum of two consecutive integers. 1

2019

11. Let n be a positive integer.

(a) Find a counterexample to show that the following statement is false.

$n^2 + n + 1$ is always a prime number. 1

(b) (i) Write down the contrapositive of:

If $n^2 - 2n + 7$ is even then n is odd. 1

(ii) Use the contrapositive to prove that if $n^2 - 2n + 7$ is even then n is odd. 3

Answers

2016

10. $p = 7 \quad = 15$ therefore not prime.

$n = 3a + 1$

$n^3 = 27a^3 + 27a^2 + 9a + 1$

$n^3 = 3(9a^3 + 9a^2 + 3a) + 1$ therefore true.

13. $n^2 = 2(2k^2 + 2k) + 1$ which is odd

•[4] contrapositive statement is true therefore original statement is true **(4)**

2018

9. a) n, $n + 1$, $n + 2$ proof

b) $n\,(n + 1)$ proof

2019

11. a) eg when $n = 4$, $n^2 + n + 1 = 21$ which is not prime

b) If n is even then $n^2 - 2n + 7$ is odd

$n = 2k,\ k \in \mathbb{N}$ and

$(2k)^2 - 2(2k) + 7$

eg $2(2k^2 - 2k + 3) + 1$ which is odd since $2k^2 - 2k + 3 \in \mathbb{N}$

contrapositive statement is true

AND

therefore original statement is true