2001

A9.

(a) Given that $-1 = \cos\theta + i\sin\theta,\ -\pi < \theta \le \pi,$ state the value of θ. 1 mark

(b) Use de Moivre's Theorem to find the non-real solutions z_1 and z_2, of the equation $z^3 + 1 = 0$. 5 marks

Hence show that $z_1^2 = -z_2$ and $z_2^2 = -z_1$. 2 marks

(c) Plot all the solutions of $z^3 + 1 = 0$. on an Argand diagram and state their geometrical significance. 3 marks

2002

A2. Verify that ***i*** is a solution of $z^4 + 4z^3 + 3z^2 + 4z + 2 = 0$. Hence find all the solutions. 5 marks

2003

A4. Identify the locus in the complex plane given by $|z + i| = 2$. 3 marks

A9. Given that $w = \cos\theta + i\sin\theta$ show that $\frac{1}{w} = \cos\theta - i\sin\theta$. 1 mark

Use de Moivre's theorem to prove $w^k + w^{-k} = 2\cos k\theta$ where *k* is a natural number. 3 marks

Expand $(w + w^{-1})^4$ by the binomial theorem and hence show that $\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$.

5 marks

2004

4. Given $z = 1 + 2i$, express $z^2 (z + 3)$ in the form $a + ib$. 2 marks
Hence or otherwise, verify that $1 + 2i$ is a root of the equation $z^3 + 3z^2 - 5z + 25 = 0$. 2 marks
Obtain the other roots of this equation. 2 marks

2005

9. Given the equation $z + 2i\bar{z} = 8 + 7i$, express z in the form $a + ib$. 4 marks

12. Let $z = \cos\theta + i\sin\theta$.

a) Use the binomial expansion to express z^4 in the form $u + iv$, where u and v are expressions involving sin θ and cosθ. 3 marks

b) Use de Moivre's theorem to write down a second expression for z^4. 1 mark

c) Using the results of (a) and (b), show that $\frac{\cos 4\theta}{\cos^2\theta} = p\cos^2\theta + q\sec^2\theta + r$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, stating the values of *p, q* and *r*.

6 marks

2006

3. Express the complex number $z = -i + \frac{1}{1-i}$ in the form $z = x + iy$ stating the values of *x* and *y*. 3 marks
Find the modulus and argument of *z* and plot *z* and $\bar{z}$ on an Argand diagram. 4 marks

2007

3. Show that $z = 3 + 3i$ is a root of the equation $z^3 - 18z + 108 = 0$ and obtain the remaining roots of the equation.

4 marks

2008

16. Given $z = \cos\theta + i\sin\theta$, use de Moivre's theorem to write down an expression for z^k in terms of θ, where k is a positive integer.

3 marks

Hence show that $\frac{1}{z^k} = \cos k\theta - i\sin k\theta$

2 marks

Deduce expressions for cos kθ and sin kθ in terms of z.

3 marks

Show that $\cos^2\theta\sin^2\theta = -\frac{1}{16}\left(z^2 - \frac{1}{z^2}\right)^2$.

2 marks

Hence show that $\cos^2\theta\sin^2\theta = a + b\cos 4\theta$, for suitable constants a and b.

2009

6. Express $z = \frac{(1+2i)^2}{7-i}$ in the form $a + ib$ where a and b are real numbers.

6 marks

Show z on an Argand diagram and evaluate $|z|$ and arg (z).

2010

16. Given $z = r(\cos\theta + i\sin\theta)$, use de Moivre's theorem to express z^3 in polar form. 1 mark

Hence obtain $\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^3$, in the form $a + ib$.

2 marks

Hence, or otherwise, obtain the roots of the equation $z^3 = 8$ in Cartesian form. 4 marks

Denoting the roots of $z^3 = 8$ by z_1, z_2, z_3:

(a) state the value $z_1 + z_2 + z_3$;

(b) obtain the value of $z_1^6 + z_2^6 + z_3^6$.

3 marks

2011

10. Identify the locus in the complex plane given by $|z-1| = 3$.

Show in the diagram the region given by $|z-1| \le 3$.

5 marks

2012

3. Given that $(-1+2i)$ is root of the equation $z^3 + 5z^2 + 11z + 15 = 0$,

4 marks

obtain all the roots. Plot all the roots on an Argand diagram.

2 marks

2013

7. Given that $z = 1 - \sqrt{3}i$, write down $\bar{z}$ and express $\bar{z}^{-2}$ in polar form. 4 marks

10. Describe the loci in the complex plane given by

(a) $|z + i| = 1$; 2 marks

(b) $|z - 1| = |z + 5|$. 3 marks

2014

16. (a) Express -1 as a complex number in polar form and hence determine the solutions to the equation $z^4 + 1 = 0$. 3 marks

(b) Write down the four solutions to the equation $z^4 - 1 = 0$. 2 marks

(c) Plot the solutions of both equations on an Argand diagram. 1 mark

(d) Show that the solutions of $z^4 + 1 = 0$ and the solutions of $z^4 - 1 = 0$ are also solutions of the equation $z^8 - 1 = 0$ 2 marks

(e) Hence identify all the solutions to the equation $z^6 + z^4 + z^2 + 1 = 0$. 2 marks

2015

13. By writing z in the form $x + iy$:

a) solve the equation $z^2 = |z|^2 - 4$; 3 marks

b) find the solutions to the equation $z^2 = i\left(|z|^2 - 4\right)$. 4 marks

2016

8. Let $z = \sqrt{3} - i$.

a) Plot z on an Argand diagram. **1 mark**

b) Let $w = az$ where $a > 0$, $a \in$ ~. Express w in polar form. **2 marks**

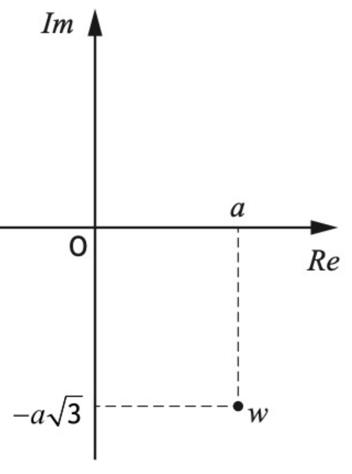
c) Express w^8 in the form $ka^n\left(x + i\sqrt{y}\right)$ where $k, x, y \in \mathbb{Z}$. **3 marks**

2018

10. Given $z = x + iy$, sketch the locus in the complex plane given by $|z| = |z - 2 + 2i|$. 3

2019

18. The complex number w has been plotted on an Argand diagram, as shown below.



(a) Express w in

(i) Cartesian form **1**

(ii) polar form. **3**

(b) The complex number z_1 is a root of $z^3 = w$, where

$$z_1 = k\left(\cos\frac{\pi}{m} + i\sin\frac{\pi}{m}\right)$$

for integers k and m.

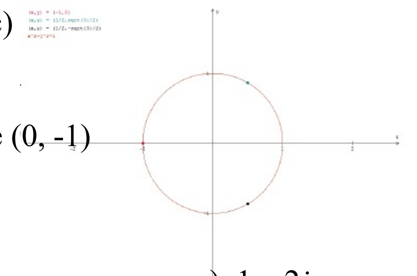
Given that $a = 4$,

(i) use de Moivre's theorem to obtain the values of k and m, and **4**

(ii) find the remaining roots. **2**

Answers

2001 A9. a) $\theta = \pi$ b) Proof c)



2002 $z = -2 \pm \sqrt{2}$

2003 A4 $x^2 + (1+y)^2 = 4$ which is a circle, centre (0, -1) radius 2.

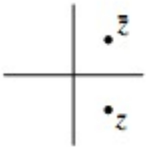
A9 Proofs

2004 4. a) $-20 + 10i$ b) Proof c) $1 - 2i$ $z = -5$

2005 9. $z = 2 + 3i$ 12a) $\left(\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta\right) + i\left(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta\right)$

b) $\cos 4\theta + i\sin 4\theta$ c) $p = 8$, $q = 1$ and $r = -8$

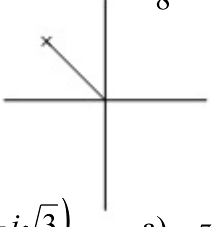
2006 3a) $\frac{1}{2} - \frac{1}{2}i$ b) $|z| = \frac{1}{2}\sqrt{2}$; $\arg z = \frac{-\pi}{4}$ or $\frac{7\pi}{4}$



2007 3a) Proof b) $3 - 3i$ and -6

2008 16a) $\cos k\theta - i\sin k\theta$ b) Proof c) Proof d) $a = \frac{1}{8}$ and $b = \frac{1}{8}$

2009 $-\frac{1}{2} + \frac{1}{2}i$ $|z| = \frac{1}{2}\sqrt{2}$ $\arg z = \frac{3\pi}{4}$



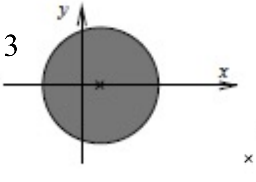
2010 16. $z^3 = r^3(\cos 3\theta + i\sin\theta)$ $2, \left(-1 + i\sqrt{3}\right), \left(-1 - i\sqrt{3}\right)$

$a = 1, \quad b = 0$

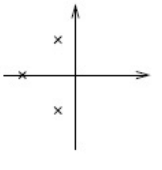
a) $z_1 + z_2 + z_3 = 0$

b) 192

2011 10. Circle Centre (1,0) radius 3



2012 3. $roots(-1+2i), (-1-2i)$ and -3.



2013 $4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

10.a) $z + i = x + iy + i = x + i(y+1)$

$|x + (y+1)i|^2 = 1$

$x^2 + (y+1)^2 = 1$

Circle centre $(0,-1)$ rad $= 1$

10.b) $|z-1|^2 = |x+5|^2$

$|(x-1) + iy|^2 = |(x+5) + iy|^2$

$(x-1)^2 + y^2 = (x+5)^2 + y^2$

$-2x + 1 = 10x + 25$

$-12x = 24$

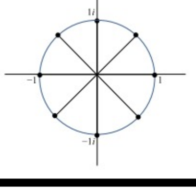
$x = -2$ which is a straight line

2014

16. $a) z = \cos\left(\frac{\pi}{4}\right) \pm i\sin\left(\frac{\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right) \pm i\sin\left(\frac{3\pi}{4}\right)$

$b) z = \pm i, \pm 1$

c)



$d) z^8 - 1 = \left(z^4+1\right)\left(z^4-1\right)$

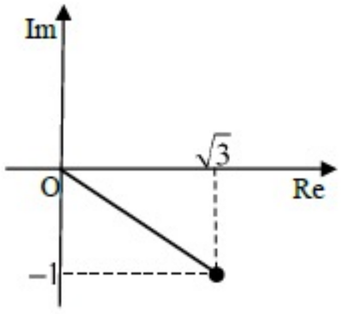
Then the solutions to $z^4 + 1 = 0$ and $z^4 - 1 = 0$ are also the solutions to $z^8 - 1 = 0$.

$e) z^8 - 1 = \left(z^4+1\right)\left(z^2+1\right)\left(z^2-1\right)$

Six solutions are those above except $z = \pm 1$

2015 13. $a)\ z = \pm\sqrt{2}i$ $\quad b)\ z = 1 - i \quad z = -1 + i$

2016 8.

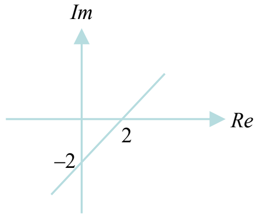


$$w = 2a\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$$

$$w^8 = 128a^8\left(-1 + i\sqrt{3}\right)$$

2018

10.



$y = x - 2$

2019

18. a) $a - a\sqrt{3}i$

$2a$

$-\frac{\pi}{3}$

$2a\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

b) $z_1 = 8^{\frac{1}{3}}\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)^{\frac{1}{3}}$

stated or implied by $\bullet^6$

$z_1 = 8^{\frac{1}{3}}\left(\cos\left(-\frac{\pi}{9}\right) + i\sin\left(-\frac{\pi}{9}\right)\right)$

$k = 2$

$m = -9$

$\ldots \pm \frac{2\pi}{3}$ stated or implied by $\bullet^{10}$

$z_2 = 2\left(\cos\frac{5\pi}{9} + i\sin\frac{5\pi}{9}\right)$

$z_3 = 2\left(\cos\left(-\frac{7\pi}{9}\right) + i\sin\left(-\frac{7\pi}{9}\right)\right)$