2001

A6. Expand $\left(x^2-\frac{2}{x}\right)^4, x\neq 0$ and simplify as far as possible. **(5)**

2002

A2. Verify that ***i*** is a solution of $z^4+4z^3+3z^2+4z+2=0$. Hence find all the solutions. **(5)**

2004

2. Obtain the binomial expansion of $(a^2-3)^4$. **(3)**

4. Given $z = 1 + 2i$, express $z^2 (z + 3)$ in the form $a + ib$. **(2)**
Hence or otherwise, verify that $1 + 2i$ is a root of the equation $z^3 + 3z^2 – 5z + 25 = 0$. **(2)**
Obtain the other roots of this equation. **(2)**

2005

9. Given the equation $z+2i\bar{z}=8+7i,$ express z in the form $a + ib$. **(4)**

2006

3. Express the complex number $z=-i+\frac{1}{1-i}$ in the form $z=x+iy$ stating the values of x and y. **(3)**
Find the modulus and argument of z and plot z and $\bar{z}$ on an Argand diagram. **(4)**

2007

1. Express the binomial expansion of $\left(x-\frac{2}{x}\right)^4$ in the form $ax^4+bx^2+c+\frac{d}{x^2}+\frac{e}{x^4}$ for integers a, b, c, d and e. **(4)**

3. Show that $z=3+3i$ is a root of the equation $z^3-18z+108=0$ and obtain the remaining roots of the equation. **(4)**

2008

8. Write down and simplify the general term in the expansion of $\left(x^2+\frac{1}{x}\right)^{10}$. **(3)**
Hence, or otherwise, obtain the term in x^{14}. **(2)**

2009

6. Express $z=\frac{(1+2i)^2}{7-i}$ in the form $a + ib$ where a and b are real numbers.

Show z on an Argand diagram and evaluate $|z|$ and arg (z). **(6)**

8. (a) Write down the binomial expansion of $(1 + x)^5$. **(1)**
(b) Hence show that $(0.9)^5$ is 0.59049. **(2)**

2011

2. Use the binomial theorem to expand $\left(\frac{1}{2}x-3\right)^4$ and simplify your answer. **(3)**

2012

3. Given that $(-1+2i)$ is root of the equation $z^3+5z^2+11z+15=0$, **(4)**

obtain all the roots. Plot all the roots on an Argand diagram. **(2)**

4. Write down and simplify the general term in the expansion of $\left(2x-\frac{1}{x^2}\right)^9$. **(3)**
Hence, or otherwise, obtain the term independent of x.

2013

1. Write down the binomial expansion of $\left(3x-\frac{2}{x^2}\right)^4$ and simplify your answer. **(4)**

7. Given that $z=1-\sqrt{3}i$, write down $\bar{z}$ and express $\left(\bar{z}\right)^2$ in polar form. **(4)**

2014

2. Write down and simplify the general term in the expression $\left(\frac{2}{x}+\frac{1}{4x^2}\right)^{10}$.

Hence, or otherwise, obtain the term in $\frac{1}{x^{13}}$. **(5)**

2015

1. Use the binomial theorem to expand and simplify $\left(\frac{x^2}{3}-\frac{2}{x}\right)^5$. **(4)**

13. By writing z in the form $x + iy$:

a) solve the equation $z^2=|z|^2-4$; **(3)**

b) find the solutions to the equation $z^2=i\left(|z|^2-4\right)$. **(4)**

2016

3. Write down and simplify the general term in the binomial expansion of $\left(\frac{3}{x}-2x\right)^{13}$.

Hence or otherwise find the term in x^9. **5 marks**

2017

1. Write down the binomial expansion of $\left(\frac{2}{y^2}-5y\right)^3$ and simplify your answer. **4 marks**

17. The complex number $z = 2 + i$ is a root of the polynomial equation $z^4 - 6z^3 + 16z^2 - 22z + q = 0$ where $q \in Z$.

a) State a second root of the equation. **1 mark**

b) Find the value of q and the remaining roots. **6 marks**

c) Show the solutions to $z^4 - 6z^3 + 16z^2 - 22z + q = 0$ on an Argand diagram. **1 mark**

2018

3. (a) Write down and simplify the general term in the binomial expansion of $\left(2x+\frac{5}{x^2}\right)^9$. 3

(b) Hence, or otherwise, find the term independent of x. 2

4. Given that $z_1 = 2+3i$ and $z_2 = p-6i$, $p \in \mathbb{R}$, find:

(a) $z_1\overline{z}_2$; 2

(b) the value of p such that $z_1\overline{z}_2$ is a real number. 1

2019

9. (a) Write down and simplify the general term in the binomial expansion of $\left(2x^2-\frac{d}{x^3}\right)^7$, where d is a constant. 3

(b) Given that the coefficient of $\frac{1}{x}$ is $-70\,000$, find the value of d. 2

Answers

2001 A6. $x^8 - 8x^5 + 24x^2 - \frac{32}{x} + \frac{16}{x^4}$

2002 A2. Proof

2004 2. $a^8 - 12a^6 + 54a^4 - 108a^2 + 81$ 4. a) $-20 + 10i$ b) Proof c) $1 - 2i$ $z = -5$

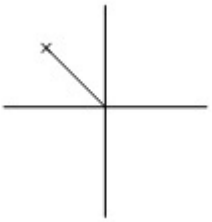
2005 9. $z = 2 + 3i$

2006 3a) $\frac{1}{2} - \frac{1}{2}i$ b) $|z| = \frac{1}{2}\sqrt{2}$; $\arg z = \frac{-\pi}{4}$ or $\frac{7\pi}{4}$

2007 1. $x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$ 3a) Proof b) $3 - 3i$ and -6

2008 8. $\binom{10}{r} x^{20-3r}$ $45x^{14}$

2009 6. $-\frac{1}{2} + \frac{1}{2}i$ $|z| = \frac{1}{2}\sqrt{2}$ $\arg z = \frac{3\pi}{4}$

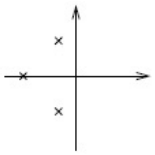


8. $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

$1 - 0.5 + 0.1 - 0.01 + 0.0005 - 0.00001 = 0.59049$

2011 2. $\frac{1}{16}x^4 - \frac{3}{2}x^3 + \frac{27}{2}x^2 - 54x + 81$

2012 3. $roots(-1+2i), (-1-2i)$ and -3.



4. $r = 3$

coefficient $= -5376$

2013 1. $81x^4 - 216x + \frac{216}{x^2} - \frac{96}{x^5} + \frac{16}{x^8}$ 7. $4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

2014 2. $240x^{-13}$ or $\frac{240}{x^{13}}$

2015 1. $\frac{x^{10}}{243} - \frac{10}{81}x^7 + \frac{40}{27}x^4 - \frac{80}{9}x + \frac{80}{3x^2} - \frac{32}{x^5}$

13. $z = \pm\sqrt{2}$ $z = 1 - i$ $z = -1 + i$

2016 3. $-1437696x^9$

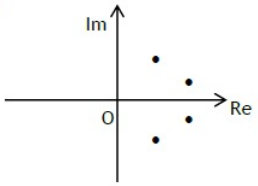
2017

1. $\frac{8}{y^6}-\frac{60}{y^3}+150-125y^3$ **(4)**

17. $2-i$

$q=15$

$1\pm\sqrt{2}\,i$



(8)

2018 3. a) $\begin{bmatrix}9\\r\end{bmatrix}(2)^{9-r}(5)^r x^{9-3r}$ b) $r=3,\ 672000$ 4. a) $2p-18+i(12+3p)$ b) $p=-4$

2019

a) $\binom{7}{r}\left(2x^2\right)^{7-r}\left(\frac{-d}{x^3}\right)^r$

x^{14-5r} or $2^{7-r}(-d)^r$

$\binom{7}{r}2^{7-r}(-d)^r x^{14-5r}$

b) $r=3$

$d=5$