# Advanced Higher Maths – Number Theory #1

1. Use the Euclidean algorithm to find the greatest common division of $312$ and $270$

2. (a) Find integers a and b so that $32a + 23b = 1$

   (b) Find integers a and b so that $50a + 123b = 1$

3. Express $106$ in base 3

4. Express $1234_5$ in decimal

5. Express $503_6$ in base 3

6. [Extension] (a) Simplify this continued fraction $3 + \frac{1}{7+\frac{1}{15}}$

   [Extension] (b) Hence express $\frac{123}{50}$ as a continued fraction (hint: use Q2b)

# Advanced Higher Maths – Number Theory #1 – Solutions

1. Euclidean algorithm

$$312 = 1.270 + 1.42$$
$$270 = 6.42 + 18$$
$$42 = 2.18 + 6$$
$$18 = 3.6 + 0$$

Hence the greatest common divisor of 312 and 270 is 6

# Advanced Higher Maths – Number Theory #1 – Solutions

2. (a) Euclidean algorithm

$$32 = 1.23 + 9$$
$$23 = 2.9 = 5$$
$$9 = 1.5 + 4$$
$$5 = 1.4 + 1$$

Now work backwards up the chain

$$1 = 1.5 - 1.4$$
$$1 = 1.5 - (9 - 1.5)$$
$$1 = -1.9 + 2.5$$
$$1 = -1.9 + 2.(23 - 2.9)$$
$$1 = 2.23 - 5.9$$
$$1 = 2.23 - 5.(32 - 1.23)$$
$$1 = -5.32 + 7.23$$

Hence $a = -5, b = 7$

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2. (b) Euclidean algorithm

$$123 = 2.50 + 23$$
$$50 = 2.23 = 4$$
$$23 = 5.4 + 3$$
$$4 = 1.3 + 1$$

Now work backwards up the chain

$$1 = 1.4 - 1.3$$
$$1 = 1.4 - (23 - 5.3)$$
$$1 = -1.23 + 6.4$$
$$1 = -1.23 + 6.(50 - 2.23)$$
$$1 = 6.50 - 13.23$$
$$1 = 6.50 - 13.(123 - 2.50)$$
$$1 = -13.123 + 32.50$$

Hence $a = -13\ b = 32$

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3. 106 in base 3

$$106 = 1(81) + 0(27) + 2(9) + 2(3) + 1$$
$$106 = 10221_3$$

4. $1234_5$ in decimal

$$1(5^3) + 2(5^2) + 3(5) + 4$$
$$= 194$$

5. $503_6$ in base 10 first

$$5(6^2) + 0(6) + 3 = 183$$

Now write 183 in base 3

$$= 2(81) + 0(27) + 2(9) + 1(3) + 0$$
$$= 20210_3$$

# Advanced Higher Maths – Number Theory #1 – Solutions

6. (a) $3+\frac{1}{7+\frac{1}{15}}=\frac{333}{106}$

(b) From part (b) we had with the Euclidan Algorithm

$$123=\mathbf{2}.50+23$$
$$50=\mathbf{2}.23=4$$
$$23=\mathbf{5}.4+3$$
$$4=\mathbf{1}.3+1$$
$$3=\mathbf{3}.1+0$$

Hence the continued fraction is

$$\frac{123}{50}=\mathbf{2}+\frac{1}{\mathbf{2}+\frac{1}{\mathbf{5}+\frac{1}{\mathbf{1}+\frac{1}{\mathbf{3}}}}}$$