Properties of Functions & Curve Sketching

Domain & Range

Questions

1. Write down the largest suitable domain for

(a) $f(x) = \sin x$ (b) $f(x) = \sqrt{x-2}$ (c) $f(x) = x!$ (d) $f(x) = \tan x$

(e) $f(x) = x^2$ (f) $f(x) = 1 + \sin x$ (g) $f(x) = \sqrt{x^2}$ (h) $f(x) = \dfrac{1}{\sin x}$

2. State the corresponding range for each of the functions in Q1 where possible.

3. Calculate the range for

$$f(x) = \frac{7}{x^2 + x - 6}$$

Answers

1. (a) $x \in \Re$ (b) $x \geq 2$ (c) $x \in W$

(d) $x \in \Re : x \neq \dfrac{(2k-1)\pi}{2}$, where $k \in Z$

(e) $x \in \Re$ (f) $x \in \Re$ (g) $x \in \Re$

(h) $x \in \Re : x \neq k\pi$, where $k \in Z$

2. (a) $f(x) \in [-1,1]$ (b) $f(x) \geq 0$ (c) $f(x) \in W$ (but not all values)

(d) $f(x) \in \Re$ (e) $f(x) \geq 0$ (f) $f(x) \in [0,2]$

(g) $f(x) \geq 0$ (h) $f(x) \geq 1$ or $f(x) \leq -1$

3. $f(x) \leq -\dfrac{28}{25}$ or $f(x) > 0$

Properties of Functions & Curve Sketching
Related Curves, Modulus and Odd & Even Functions

QUESTION

1. Write down the equations of the inverses of the following functions :

(a) $f(x) = 2x$ (b) $f(x) = 2 - x$ (c) $f(x) = {}^{2}/_{x}$

(d) $f(x) = 2^x$ (e) $f(x) = 1 - 2x$ (f) $f(x) = \ln(x - 2)$

2. Evaluate :

(a) $\sin^{-1}({}^{\sqrt{3}}/_{2})$ (b) $\tan^{-1}({}^{1}/_{\sqrt{3}})$ (c) $\tan^{-1}(1)$

(d) $\sin^{-1}({}^{1}/_{2})$ (e) $\cos^{-1}(-{}^{\sqrt{3}}/_{2})$ (f) $\tan^{-1}(\sqrt{3})$

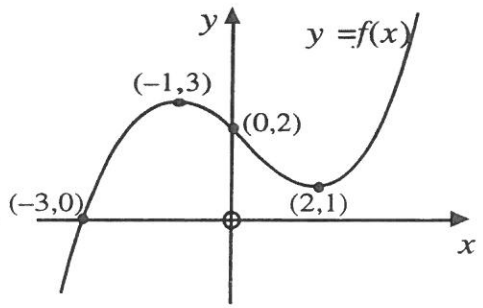
3. Sketch the following graph of a function $y = f(x)$ for each part of the following question.

On separate graphs, sketch the graphs of :

(a) $f(x - 3)$ (b) $f(x + 3)$

(c) $f(x) + 2$ (d) $2f(x)$

(e) $-f(x)$ (f) $f(-x)$



4. Sketch the graphs of $f(x)$ and $|f(x)|$.

(a) $f(x) = x + 2$ (b) $f(x) = 5 - 2x$ (c) $f(x) = x^2 - 2x - 3$

(d) $f(x) = 3x - x^2$ (e) $f(x) = x^3 + 1$ (f) $f(x) = \dfrac{1}{x} - 2$

5. Which of the following functions are odd, even or neither ?

(a) $f(x) = (x + 4)(x - 2)$ (b) $f(x) = 3x^2 + 5$

(c) $f(x) = 2x - x^3$ (d) $f(x) = \sin 2x$

Properties of Functions & Curve Sketching
Related Curves, Modulus and Odd & Even Functions

ANSWER!

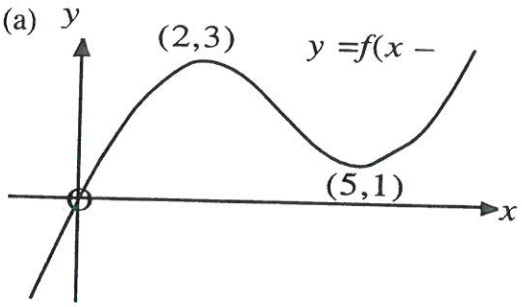
1. (a) $^1/_2x$ (b) $2 - x$ (c) $^2/_x$

   (d) $\log_2 x$ (e) $1 - x$ (f) $10^x + 2$

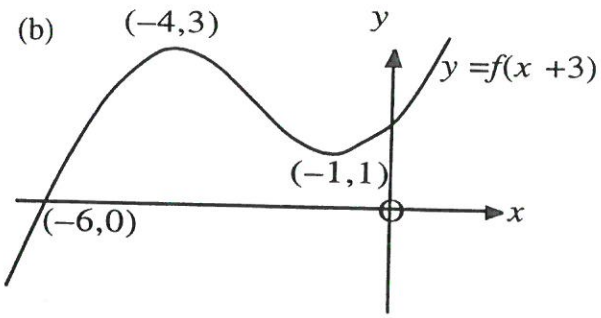
2. (a) $\pi/3$ (b) $\pi/6$ (c) $\pi/4$

   (d) $\pi/6$ (e) $-\pi/6$ (f) $\pi/6$

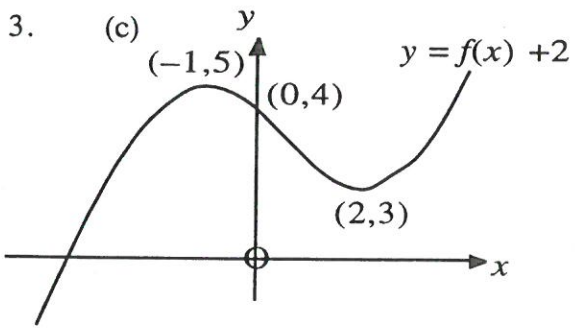
3. (a)



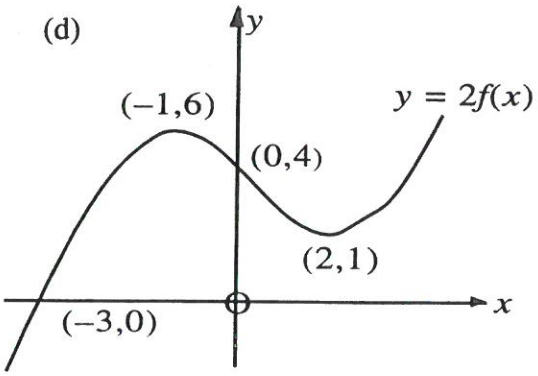
(b)

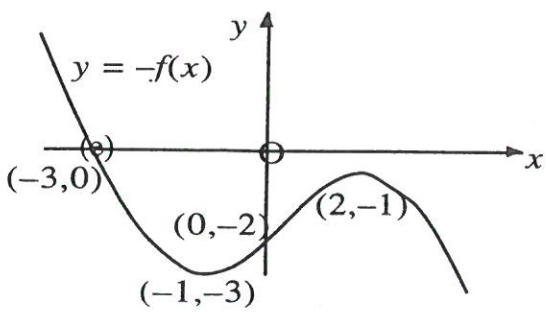


3. (c)



(d)





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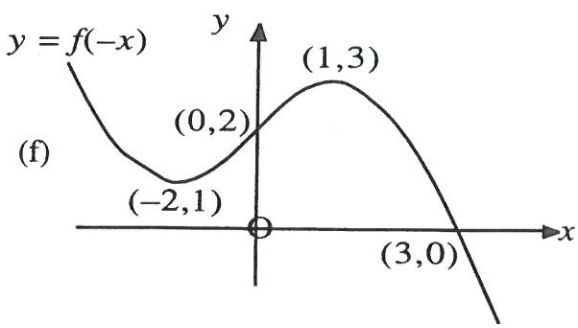
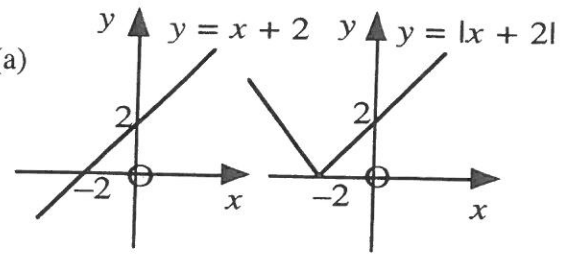
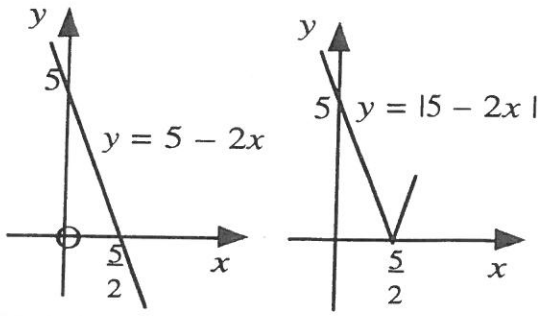


Properties of Functions & Curve Sketching
Related Curves, Modulus and Odd & Even Functions

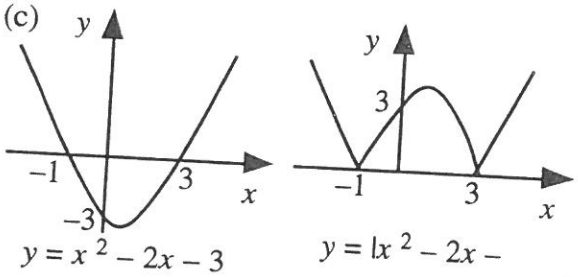
4. (a)



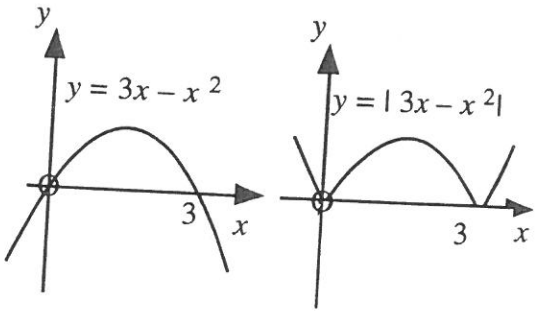
(b)



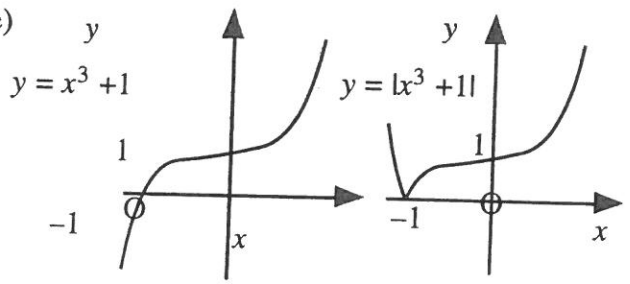
(c)



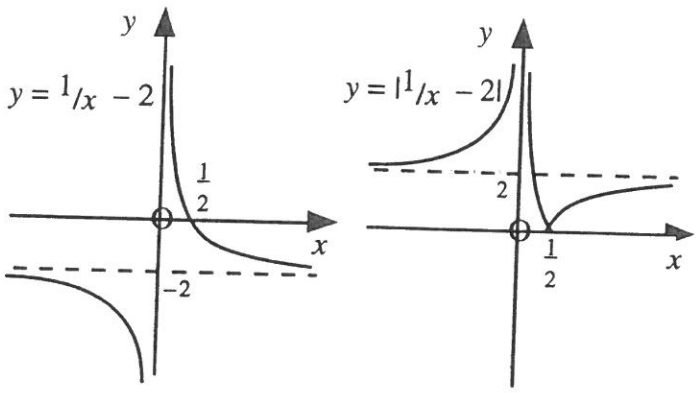
(d)



(e)



(f)



5. (a) neither (b) even (c) odd
(d) odd (e) even (f) even

Properties of Functions & Curve Sketching
Vertical Asymptotes

QUESTION

Find all the vertical asymptotes of the following rational functions.

1. $f(x)=\dfrac{4}{x-2}$
2. $f(x)=\dfrac{3x-1}{x^2+2x-3}$
3. $f(x)=\dfrac{12}{x^2-2x-3}$
4. $f(x)=\dfrac{x+4}{x-2}$
5. $f(x)=\dfrac{x^2}{4-x^2}$
6. $f(x)=\dfrac{x(x+1)}{(x-1)(x+2)}$
7. $f(x)=\dfrac{(x-1)(x-4)}{x}$
8. $f(x)=\dfrac{x^2+3}{x-1}$
9. $f(x)=\dfrac{x^3}{x^2+3}$
10. $f(x)=\dfrac{x}{x^2+4}$
11. $f(x)=\dfrac{x^2}{x-1}$
12. $f(x)=\dfrac{2x^2}{x^2-1}$

ANSWERS

1. $x = 2$
2. $x = -3$ $x = 1$
3. $x = -1$ $x = 3$
4. $x = 2$
5. $x = -2$ $x = 2$
6. $x = -2$ $x = 1$
7. $x = 0$
8. $x = 1$
9. None
10. None
11. $x = 1$
12. $x = -1$ $x = 1$

Properties of Functions & Curve Sketching
Vertical Asymptotes

QUESTIONS

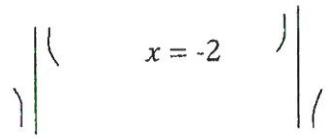
Find the equations of the vertical asymptotes to the following curves and show how the curve approaches each asymptote.

(1) $y = \dfrac{x(x-1)}{x+1}$ (2) $y = \dfrac{x^2}{4-x^2}$ (3) $y = \dfrac{(x-1)(x+2)}{x}$

Answers

(1) $x = -1$

(2) $x = 2$ $x = -2$



(3) $x = 0$

Properties of Functions & Curve Sketching
Non-Vertical Asymptotes

QUESTION

Find the non - vertical asymptote of the following rational functions.

1. $f(x)=\dfrac{4}{x-2}$
2. $f(x)=\dfrac{3x-1}{x^2+2x-3}$
3. $f(x)=\dfrac{12}{x^2-2x-3}$
4. $f(x)=\dfrac{x+4}{x-2}$
5. $f(x)=\dfrac{x^2}{4-x^2}$
6. $f(x)=\dfrac{x(x+1)}{(x-1)(x+2)}$
7. $f(x)=\dfrac{(x-1)(x-4)}{x-2}$
8. $f(x)=\dfrac{x^2+3}{x-1}$
9. $f(x)=\dfrac{x^3}{x^2+3}$
10. $f(x)=\dfrac{x}{x^2+4}$
11. $f(x)=\dfrac{x^2}{x-1}$
12. $f(x)=\dfrac{2x^2}{x^2-1}$

ANSWERS

1. $y=0$
2. $y=0$
3. $y=0$
4. $y=1$
5. $y=-1$
6. $y=1$
7. $y=x-3$
8. $y=x+1$
9. $y=x$
10. $y=0$
11. $y=x+1$
12. $y=2$

Properties of Functions & Curve Sketching
Non-Vertical Asymptotes

QUESTION

Find equations of asymptotes (vertical, horizontal and oblique)

(1) $y = \dfrac{(x+1)^3}{x^2}$ (2) $y = \dfrac{x^2}{x-1}$ (3) $y = \dfrac{x^2-10x+9}{x^2+10x+9}$

(4) $y = \dfrac{x^2-2x+9}{x-2}$ (5) $y = \dfrac{2x^2}{x^2-1}$ (6) $y = \dfrac{x^2+2x+4}{x+4}$

(7) $y = \dfrac{5}{x-3}$ (8) $y = \dfrac{x+1}{x-1}$ (9) $y = \dfrac{x^2+1}{x}$

(10) $y = \dfrac{x^3+1}{x+1}$ (11) $y = (x-1) + \dfrac{4}{x+1}$ (12) $y = \dfrac{x}{x^2+1}$

(13) $y = \dfrac{2x}{(x+1)(x-1)}$ (14) $y = \dfrac{2x}{x-3}$ (15) $y = \dfrac{3x}{x+2}$

(16) $y = \dfrac{x^3}{x^2-1}$ (17) $y = \dfrac{x^3+8}{x^2-1}$ (18) $y = \dfrac{4x}{3x^3+1}$

Answers

(1) V.A. $x = 0$
O.A. $y = x + 3$

(2) V.A. $x = 1$
O.A. $y = x + 1$

(3) H.A. $y = 1$
V.A.s $x = -1$ and $x = -9$

(4) V.A. $x = 2$
O.A. $y = x$

(5) H.A. $y = 2$
V.A.s $x = 1$ and $x = -1$

(6) O.A. $y = x - 2$
V.A. $x = -4$

(7) H.A. $y = 0$
V.A. $x = 3$

(8) V.A. $x = 1$
H.A. $y = 1$

(9) V.A. $x = 0$
O.A. $y = x$

(10) no asymptotes
(parabola)

(11) O.A. $y = x - 1$
V.A. $x = -1$

(12) H.A. $y = 0$

(13) V.A.s $x = 1$ and $x = -1$
H.A. $y = 0$

(14) V.A. $x = 3$
H.A. $y = 2$

(15) V.A. $x = -2$
H.A. $y = 3$

(16) V.A.s $x = 1$ and $x = -1$
O.A. $y = x$

(17) V.A.s $x = 1$ and $x = -1$
O.A. $y = x$

(18) V.A. $x = \dfrac{1}{\sqrt[3]{3}}$
H.A. $y = 0$

Properties of Functions & Curve Sketching

Curve Sketching

QUESTION

Sketch the following curves.

1. $f(x)=\dfrac{4}{x-2}$

2. $f(x)=\dfrac{x-2}{x-1}$

3. $f(x)=\dfrac{x^2+x-2}{x^2+x-6}$

4. $f(x)=\dfrac{x^2+2x+5}{x+1}$

5. $f(x)=\dfrac{x+1}{x^2+2x+5}$

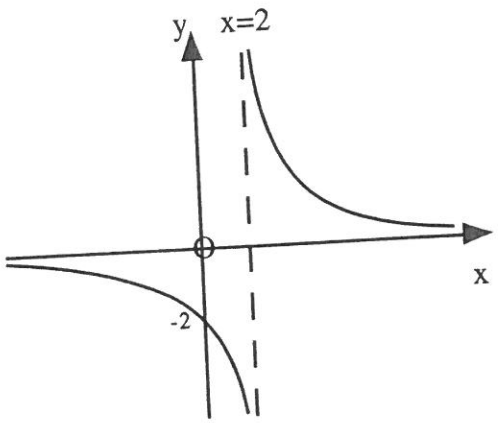
6. $f(x)=\dfrac{2x^2}{x^2-1}$

7. $f(x)=\dfrac{x^2-10x+9}{x^2+10x+9}$

8. $f(x)=\dfrac{2x^2-3x-3}{x^2-3x+2}$

ANSWER

1.



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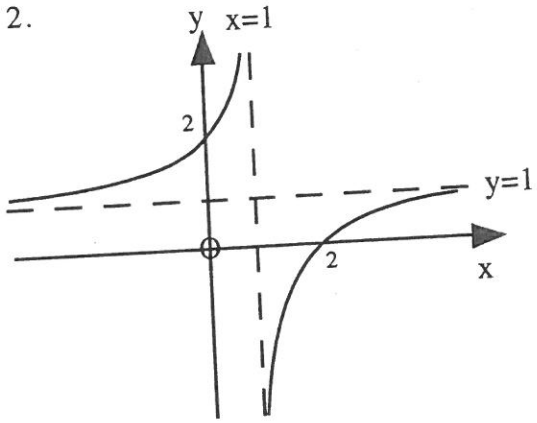

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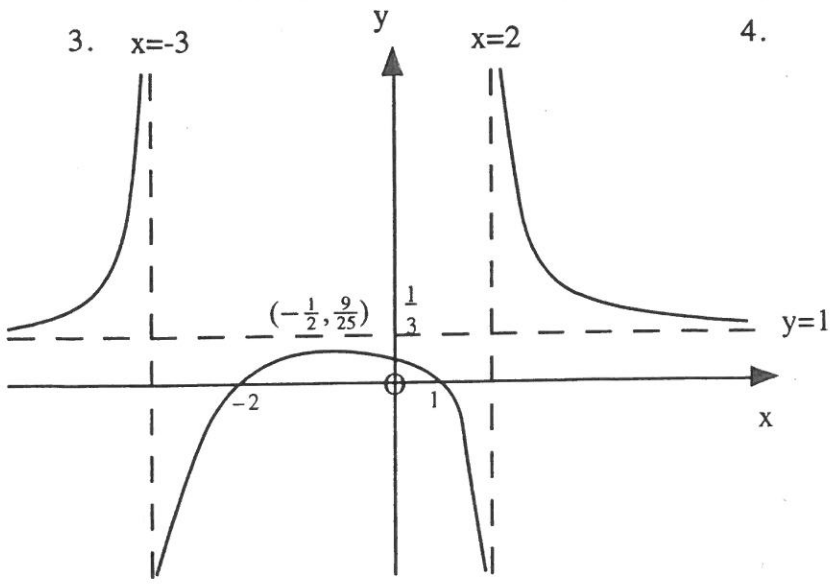

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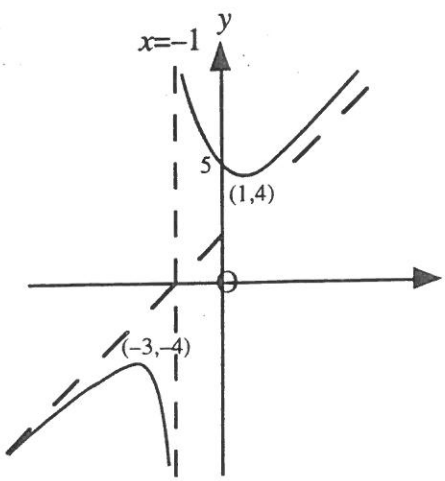
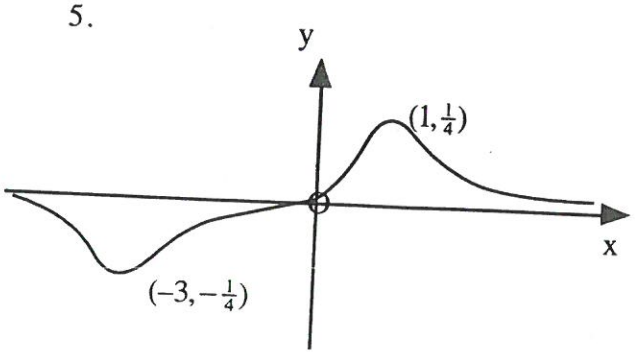


Properties of Functions & Curve Sketching

Curve Sketching

5.



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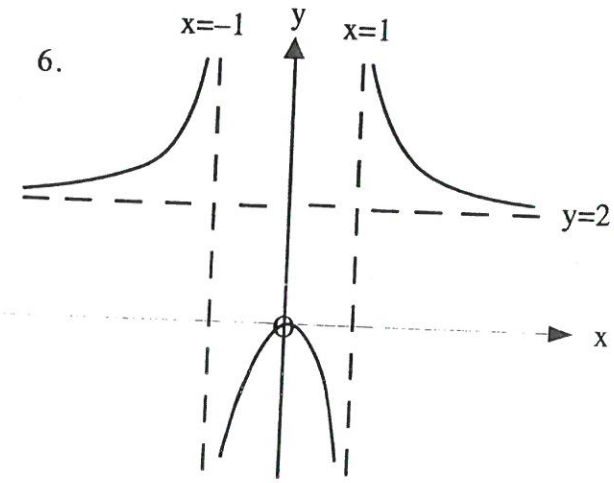

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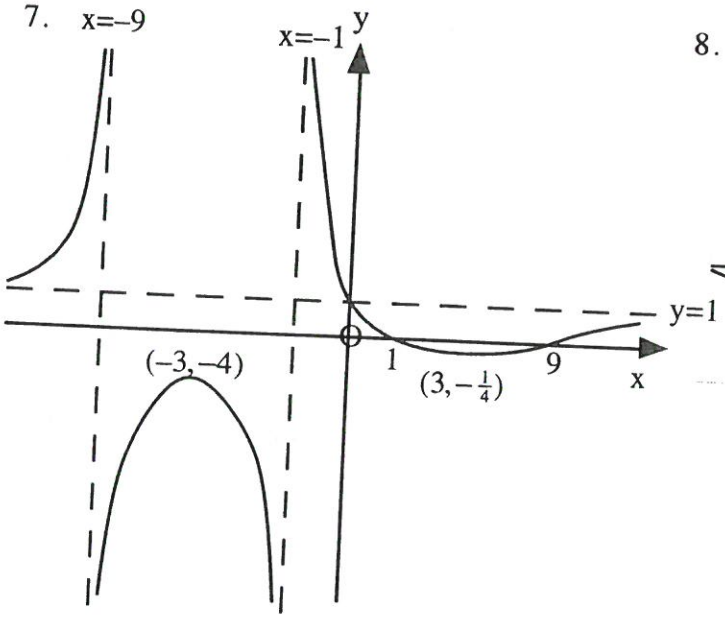

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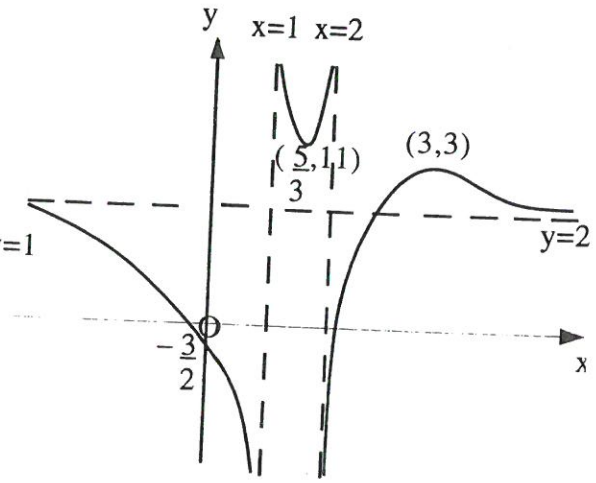


Properties of Functions & Curve Sketching
Curve Sketching

QUESTION

Sketch the following curves.

1. $y = \dfrac{4x-5}{x^2-1}$

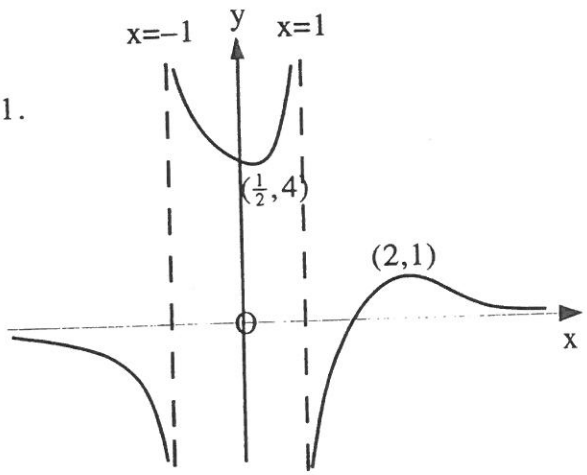
2. $y = \dfrac{x^2+3}{x-1}$

3. $y = \dfrac{4(x+1)}{x^2+2x+2}$

4. $y = \dfrac{2x-1}{(x-1)^2}$

ANSWER

1.



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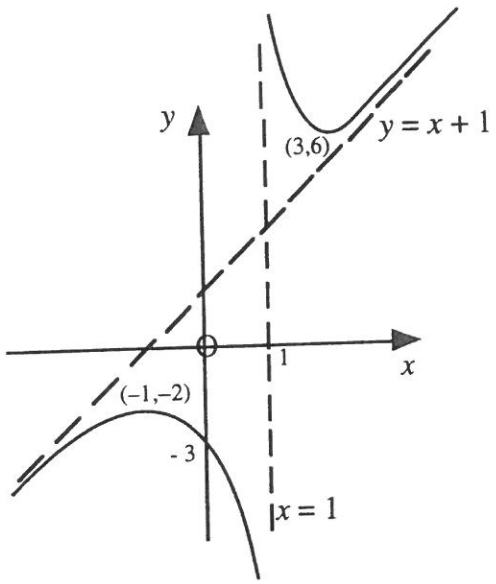

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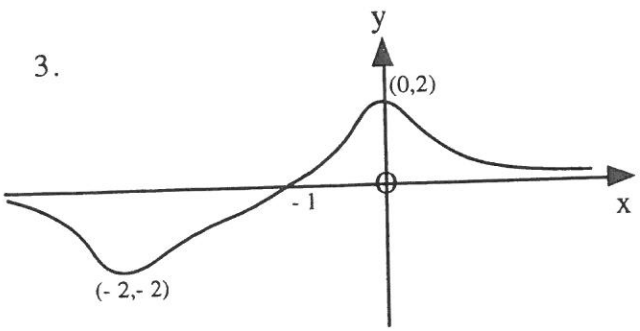

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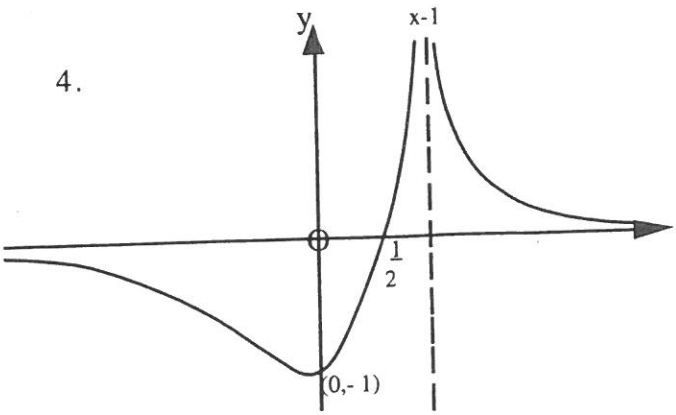

Properties of Functions & Curve Sketching
Curve Sketching

QUESTION

Sketch the curves:

(1) $y = \dfrac{x+2}{x-1}$ (2) $y = \dfrac{(x-1)(x+2)}{(x+3)(x-2)}$ (3) $y = \dfrac{x}{x^2+1}$

(4) $y = x^4 - 2x^2 - 3$ (5) $y = \dfrac{(x+1)^3}{x^2}$ (6) $y = \dfrac{x^2-2x+9}{x-2}$

(7) $y = \dfrac{2x^2}{x^2-1}$ (8) $y = \dfrac{x^2+2x+5}{x+1}$

(9) $y = \dfrac{x^2-10x+9}{x^2+10x+9}$

Answers

(1) V.A. $x = 1$, H.A. $y = 1$; (-2, 0); no stationary values

(2) V.A.s $x = -3$ and $x = 2$, H.A. $y = 1$; (1, 0), (-2, 0); max t.p. (-0.5, -0.36)

(3) No asymptotes; (0, 0); min t.p. (-1, -0.5), max t.p. (1, 0.5)

(4) No asymptotes; ($-\sqrt{3}$, 0), ($\sqrt{3}$, 0); min t.p. (-1, -4), min t.p. (1, -4), max t.p. (0, -3)

(5) V.A. $x = 0$, O.A. $y = x + 3$; (-1, 0); min t.p. (2, 6.75), pt of inflex. (-1, 0)

(6) V.A. $x = 2$, O.A. $y = x$; no zeros; min t.p. (5, 8), max t.p. (-1, -4)

(7) V.A.s $x = 1$, $x = -1$, H.A. $y = 2$; (0, 0); max t.p. (0, 0)

(8) V.A. $x = -1$, O.A. $y = x + 1$; no zeros; min t.p. (1, 4), max t.p. (-3, -4)

(9) V.A.s $x = -9$, $x = -1$, H.A. $y = 1$; (1, 0), (9, 0); min t.p. (3, -0.25), max t.p. (-3, -4)

The shapes of the graphs can be checked using a graphic calculator.

Properties of Functions & Curve Sketching
Points of Inflexion

QUESTION

Find the points of inflexion of the curves and determine the shape of each curve in a neighbourhood of each of the points.

(1) $y = 3x - x^3$ (2) $y = x^4 + 4x^3$ (3) $y = x^5 + x^3 + x$

(4) $y = 3x^4 - 8x^3 + 6x^2$ (5) $y = \dfrac{x}{x^4 + 3}$ (6) $y = \dfrac{4x}{3x^2 + 1}$

Answers

(1) (0, 0) shape: +++ (2) (0, 0) shape: +0+ , (-2, -16) shape: +++

(3) (0, 0) shape: +++ (4) $(\frac{1}{3}, \frac{11}{27})$ shape: +++, (1, 0) shape +0+

(5) $(-\sqrt{2}, \frac{\sqrt{2}}{7})$ shape: ---, $(\sqrt{2}, \frac{-\sqrt{2}}{7})$ shape: ---

(6) (-1, -1) shape: ---, (0, 0) shape: +++ , (1, 1) shape: ---

QUESTION

Find any points of inflexion on:

(1) $y = 3x^3 - 4x + 5$ (2) $y = x^4 - 6x^2 - 8x$

(3) $y = x(x^2 - 1)$ (4) $y = x^2(x + 6)$

Answers

(1) (0, 5) (2) (-1, 3), (1, -13) (3) (0, 0) (4) (-2, 16)