

Proof by Induction Practice

Questions

Prove by Induction:-

1. $\sum_{r=1}^n \frac{1}{r(r+1)} = \frac{n}{n+1}$, $n \geq 1$ 2. $\sum_{r=1}^n (2r-1) = (n-2)(n+2)$, $n \geq 3$

3. $(\cos \theta + \sin \theta)^n = \cos n\theta + \sin n\theta$

4. The sum of the first n odd numbers is a perfect square. (i.e. $\sum_{k=1}^n 2k-1 = p^2$)

5. $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$.

6. $\sum_{r=1}^n r(r+1)(r+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$.

7. S_n of the series $\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots = \frac{n}{2n+1}$

8. S_n of the series $\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \dots = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

9. $n^3 + 3n^2 - 10n$ is divisible by 3.

10. $7^n + 4^n + 1^n$ is divisible by 6.

11. For all integers $n > 2$, $2^n > 2n$.

12. For all integers $n \geq 4$, $3^n > n^3$.

ANSWERS:

PROOF

Proof by Induction Practice

QUESTIONS

1. Prove by induction that for $a > 0$,

$$(1 + a)^n \geq 1 + na$$

for all positive integers n .

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2. Prove by induction that $8^n + 3^{n-2}$ is divisible by 5 for all integers $n \geq 2$.

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3. Use induction to prove that

$$\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$$

for all positive integers n .

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4. Prove by induction that $4^n - 1$ is divisible by 3 for all positive integers n .

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5. Prove by induction that $\frac{d^n}{dx^n}(xe^x) = (x+n)e^x$ for all integers $n \geq 1$.

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ANSWERS:

PROVE