Complex Numbers
Basic Operations

QUESTIONS

1. Express each of the following in the form $x + yi$.

(a) $(3 + 7i) + (2 + i)$ (b) $(9 - 2i) - (3 + i)$

(c) $(-2 + i) + (7 - 4i)$ (d) $(3 + 2i) + (3 - 2i)$

(e) $(-2 + i) - (-2 - i)$ (f) $(a + bi) + (a - bi)$

(g) $(a + bi) - (a - bi)$

2. Using the fact that $i^2 = -1$,
express i^3, i^4, i^5, i^6, i^7, i^8, i^9 and i^{10} in their simplest form.

3. Simplify

(a) $2i \times 4i$ (b) $-2i^2$

(c) $i(3 + 2i)$ (d) $-i(1 - 4i)$

(e) $(2 + i)(3 + i)$ (f) $(6 - 5i)(2 + 3i)$

(g) $(2 + 3i)(2 - 3i)$ (h) $(a + bi)(a - bi)$

(i) $(a + bi)(c + di)$ (j) $(a + bi)(c - di)$

(k) $(1 + i)^3$ (l) $(1 + i)^4$

(m) $(1 + i)^4(1 - i)^5$ (n) $(3 + i)^2 + (3 - i)^2$

(o) $(\cos t + i\sin t)^2$

(p) $(\cos A + i\sin A)(\cos B + i\sin B)$

4. Simplify

(a) $(2 + i)(2 - i)$ (b) $(1 - 2i)(1 + 2i)$ (c) $(5 - i)(5 + i)$

5. Simplify and express in the form $x + yi$

(a) $\dfrac{4+i}{i}$ (b) $\dfrac{1}{2+i}$ (c) $\dfrac{2-i}{1-2i}$

(d) $\dfrac{5+i}{5-i}$ (e) $\dfrac{a+bi}{a-bi}$ (f) $\dfrac{a+bi}{c+di}$

(g) $\dfrac{10+5i}{2-i}$ (h) $\dfrac{1}{\cos A + i\sin A}$ (i) $\dfrac{\cos A + i\sin A}{\cos A - i\sin A}$

Complex Numbers
Basic Operations

ANSWERS

1. (a) $5 + 8i$ (b) $6 - 3i$ (c) $5 - 3i$ (d) 6
   (e) $2i$ (f) $2a$ (g) $2bi$

2. $i^3 = -i,\ i^4 = 1,\ i^5 = i,\ i^6 = -1,\ i^7 = -i,\ i^8 = 1,\ i^9 = i,\ i^{10} = -1.$

3. (a) -8 (b) 2 (c) $-2 + 3i$ (d) $-4 - i$
   (e) $5 + 5i$ (f) $27 + 8i$ (g) 13 (h) $a^2 + b^2$
   (i) $(ac - bd) + (bc - ad)i$ (j) $(ac + bd) + (bc - ad)i$
   (k) $-2 + 2i$ (l) -4 (m) $16 - 16i$ (n) 16
   (o) $\cos 2t + i\sin 2t$ (p) $\cos(A + B) + i\sin(A + B)$

4. (a) 5 (b) 5 (c) 26

5. (a) $1 - 4i$ (b) $\frac{2}{5} - \frac{1}{5}i$ (c) $\frac{4}{5} + \frac{3}{5}i$
   (d) $\frac{12}{13} + \frac{5}{13}i$ (e) $\frac{a^2 - b^2}{a^2 + b^2} + \frac{2ab}{a^2 + b^2}i$ (f) $\frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i$
   (g) $3 + 4i$ (h) $\cos A - i\sin A$ (i) $\cos 2A + i\sin 2A$

Complex Numbers
Modulus, Argument & Argand Diagrams

QUESTION

1. Find the modulus and argument of :-

(a) $1 + \sqrt{3}i$ (b) $2 - 2i$ (c) $-\sqrt{2} - \sqrt{2}i$

(d) $2i$ (e) 3 (f) $-\sqrt{3} + i$

(g) $-3i$ (h) -5 (i) $-3 - 3i$

(IN 1 ALSO DRAW THE ARGAND DIAGRAM)

2. Given that $z_1 = -3 + 3\sqrt{3}i$ and $z_2 = \sqrt{3} + i$,

(a) (i) find $|z_1|$, $|z_2|$, $|z_1 z_2|$ (ii) find $\arg(z_1)$, $\arg(z_2)$, $\arg(z_1 z_2)$

(b) Repeat for $z_1 = 3i$ and $z_2 = \sqrt{2} - \sqrt{2}i$. What do you notice?

3. Given that $z_1 = -3 + 3\sqrt{3}i$ and $z_2 = \sqrt{3} + i$,

(a) (i) find $|z_1|$, $|z_2|$, $\left|\frac{z_1}{z_2}\right|$.

(ii) find $\arg(z_1)$, $\arg(z_2)$, $\arg\left(\frac{z_1}{z_2}\right)$.

(b) Repeat for $z_1 = 3i$ and $z_2 = \sqrt{2} - \sqrt{2}i$. What do you notice?

4. Given that $z = 1 + i$

(a) (i) find $|z|$, $|iz|$ (ii) find $\arg(z)$, $\arg(iz)$

(b) Repeat for $z = -\sqrt{3} - i$. What do you notice?

5. Given that $z = 1 + i$

(a) (i) find $|z|$, $|\bar{z}|$ (ii) find $\arg(z)$, $\arg(\bar{z})$

(b) Repeat for $z = -\sqrt{3} - i$. What do you notice?

6. Given that $z = 1 + i$

(a) (i) find $z\bar{z}$ (ii) find $|z\bar{z}|$ and $\arg(z\bar{z})$

(b) Repeat for $z = -\sqrt{3} - i$. What do you notice?

Complex Numbers
Modulus, Argument & Argand Diagrams

ANSWERS

1. (a) $|z| = 2$, $\arg(z) = \frac{\pi}{3}$ (b) $|z| = 2\sqrt{2}$, $\arg(z) = \frac{-\pi}{4}$

(c) $|z| = 2$, $\arg(z) = \frac{-3\pi}{4}$ (d) $|z| = 2$, $\arg(z) = \frac{\pi}{2}$

(e) $|z| = 3$, $\arg(z) = 0$ (f) $|z| = 2$, $\arg(z) = \frac{5\pi}{6}$

(g) $|z| = 3$, $\arg(z) = -\frac{\pi}{2}$ (h) $|z| = 5$, $\arg(z) = \pi$

(i) $|z| = 3\sqrt{2}$, $\arg(z) = \frac{-3\pi}{4}$

+ CHECK DIAGRAMS

2. (a) (i) $|z_1| = 6$, $|z_2| = 2$, $|z_1 z_2| = 12$

(ii) $\arg(z_1) = \frac{2\pi}{3}$, $\arg(z_2) = \frac{\pi}{6}$, $\arg(z_1 z_2) = \frac{5\pi}{6}$

(b) (i) $|z_1| = 3$, $|z_2| = 2$, $|z_1 z_2| = 6$

(ii) $\arg(z_1) = \frac{\pi}{2}$, $\arg(z_2) = -\frac{\pi}{4}$, $\arg(z_1 i_2) = \frac{\pi}{4}$

$\therefore$ $|z_1 z_2| = |z_1| \times |z_2|$ and $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

3. (a) (i) $|z_1| = 6$, $|z_2| = 2$, $\left|\frac{z_1}{z_2}\right| = 3$

(ii) $\arg(z_1) = \frac{2\pi}{3}$, $\arg(z_2) = \frac{\pi}{6}$, $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$

(b) (i) $|z_1| = 3$, $|z_2| = 2$, $\left|\frac{z_1}{z_2}\right| = \frac{3}{2}$

(ii) $\arg(z_1) = \frac{\pi}{2}$, $\arg(z_2) = -\frac{\pi}{4}$, $\arg\left(\frac{z_1}{z_2}\right) = \frac{3\pi}{4}$

$\therefore$ $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ and $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

4. (a) (i) $|z| = \sqrt{2}$, $|iz| = \sqrt{2}$

(ii) $\arg(z) = \frac{\pi}{4}$, $\arg(iz) = \frac{3\pi}{4}$

(b) (i) $|z| = 2$, $|iz| = 2$

(ii) $\arg(z) = -\frac{5\pi}{6}$, $\arg(iz) = -\frac{\pi}{3}$

$\therefore$ $|iz| = |z|$ and $\arg(iz) = \arg(z) + \frac{\pi}{2}$

or multiplying by i rotates z by $\frac{\pi}{2}$ about the origin

Complex Numbers
Modulus, Argument & Argand Diagrams

5. (a) (i) $|z| = \sqrt{2}$, $|\bar{z}| = \sqrt{2}$

(ii) $\arg(z) = \frac{\pi}{4}$, $\arg(\bar{z}) = -\frac{\pi}{4}$

(b) (i) $|z| = 2$, $|\bar{z}| = 2$

(ii) $\arg(z) = -\frac{5\pi}{6}$, $\arg(\bar{z}) = \frac{5\pi}{6}$

$\therefore$ $|\bar{z}| = |z|$ and $\arg(\bar{z}) = -\arg(z)$
or $\bar{z}$ is a reflection of z in the real axis.

6. (a) (i) $z\bar{z} = 2$

(ii) $|z\bar{z}| = 2$, $\arg(z\bar{z}) = 0$

(b) (i) $z\bar{z} = 4$

(ii) $|z\bar{z}| = 4$, $\arg(z\bar{z}) = 0$

$\therefore$ $z\bar{z}$ is always real, $|z\bar{z}| = |z|^2$ and $\arg(z\bar{z}) = 0$

Complex Numbers
Loci

QUESTKNI

1. Find the set of points z, where $|z| = 2$.
2. Find the set of points z, where $|z - 3| = 5$.
3. Find the set of points z, where $|z + 3| = |z - 4i|$.
4. Find the set of points z, where $|z - 2| > 3$.
5. Find the set of points z, where $|z - (3 + 2i)| = 4$.
6. Find the set of points z, where $|z - 1| \leq 4$.
7. Find the set of points z, where $|z - 2| = |z + 1 - i|$.

ANSWERI

1. $x^2 + y^2 = 4$, a circle centre O, radius 2.
2. $(x - 3)^2 + y^2 = 25$, a circle centre (3, 0), radius 5.
3. $6x - 8y = 7$, the perpendicular bisector of (– 3, 0) and (0, 4).
4. $(x - 2)^2 + y^2 > 9$, outside the circle centre (2, 0), radius 3.
5. $(x - 3)^2 + (y - 2)^2 = 16$, a circle centre (3, 2), radius 4.
6. $(x - 1)^2 + y^2 \leq 16$, on or inside the circle centre (1, 0), radius 4.
7. $y = 3x - 1$, the perpendicular bisector of (2, 0) and (– 1, 1).

Complex Numbers
Polar Form

QUESTION

1. (i) For each of the following, find $|z_1|$, $|z_2|$ and $|z_1 z_2|$.

   (ii) Find also $\arg(z_1)$, $\arg(z_2)$ and $\arg(z_1 z_2)$

   (iii) Hence, find the product of $z_1 z_2$ in its simplest form each time.

   (a) $z_1 = 2[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)]$ $\quad z_2 = 3[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)]$

   (b) $z_1 = \sqrt{2}[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)]$ $\quad z_2 = 2[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)]$

   (c) $z_1 = r_1[\cos\theta_1 + i\sin\theta_1]$ $\quad z_2 = r_2[\cos\theta_2 + i\sin\theta_2]$
   This is the general result.

2. Write down, using the results from above, $|z_1 z_2|$, $\arg(z_1 z_2)$ and $z_1 z_2$:-

   (a) $z_1 = 3[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)]$ $\quad z_2 = 2[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)]$

   (b) $z_1 = \frac{1}{2}[\cos\left(\frac{2\pi}{15}\right) + i\sin\left(\frac{2\pi}{15}\right)]$ $\quad z_2 = 2[\cos\left(\frac{13\pi}{15}\right) + i\sin\left(\frac{13\pi}{15}\right)]$

   (c) $z_1 = 10[\cos\left(\frac{5\pi}{9}\right) + i\sin\left(\frac{5\pi}{9}\right)]$ $\quad z_2 = 10[\cos\left(\frac{13\pi}{9}\right) + i\sin\left(\frac{13\pi}{9}\right)]$

3. (i) Find $|z_1|$, $|z_2|$ and $\left|\frac{z_1}{z_2}\right|$ each time.

   (ii) Find $\arg(z_1)$, $\arg(z_2)$ and $\arg\left(\frac{z_1}{z_2}\right)$.

   (iii) Find the quotient of $\frac{z_1}{z_2}$ in its simplest form.

   (a) $z_1 = 2[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)]$ $\quad z_2 = 3[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)]$

   (b) $z_1 = \sqrt{2}[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)]$ $\quad z_2 = 2[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)]$

   (c) $z_1 = r_1[\cos\theta_1 + i\sin\theta_1]$ $\quad z_2 = r_2[\cos\theta_2 + i\sin\theta_2]$
   This is the general result.

Complex Numbers
Polar Form

4. Write down, using the results from earlier, $\left|\frac{z_1}{z_2}\right|$, $\arg\left(\frac{z_1}{z_2}\right)$ and $\frac{z_1}{z_2}$ each time.

(a) $z_1 = 3[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)]$ $\quad z_2 = 2[\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)]$

(b) $z_1 = 2[\cos\left(\frac{13\pi}{15}\right) + i\sin\left(\frac{13\pi}{15}\right)]$ $\quad z_2 = \frac{1}{2}[\cos\left(\frac{2\pi}{15}\right) + i\sin\left(\frac{2\pi}{15}\right)]$

(c) $z_1 = 10[\cos\left(\frac{13\pi}{9}\right) + i\sin\left(\frac{13\pi}{9}\right)]$ $\quad z_2 = 10[\cos\left(\frac{5\pi}{9}\right) + i\sin\left(\frac{5\pi}{9}\right)]$

5. Express each of the following in its simplest form, using earlier results.

(a) $[\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)] \times [\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)] \times [\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)]$

(b) $4[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)] \times 2[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)]$

(c) $20[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)] \div [\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)]$

(d) $[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)] \div 3[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)]$

ANSWERS

1. (a) (i) $|z_1| = 2, |z_2| = 3, |z_1z_2| = 6$ (ii) $\arg(z_1) = \frac{\pi}{4}, \arg(z_2) = \frac{\pi}{4}, \arg(z_1z_2) = \frac{\pi}{2}$

(iii) $6[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}]$

(b) (i) $|z_1| = \sqrt{2}, |z_2| = 2, |z_1z_2| = 2\sqrt{2}$ (ii) $\arg(z_1) = \frac{\pi}{6}, \arg(z_2) = \frac{\pi}{3}, \arg(z_1z_2) = \frac{\pi}{2}$

(iii) $2\sqrt{2}[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}]$

(c) (i) $|z_1| = r_1, |z_2| = r_2, |z_1z_2| = r_1r_2$

(ii) $\arg(z_1) = \theta_1, \arg(z_2) = \theta_2, \arg(z_1z_2) = \theta_1 + \theta_2$

(iii) $r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

2. (a) $|z_1z_2| = 6, \arg(z_1z_2) = 0, z_1z_2 = 6[\cos 0 + i\sin 0]$.

(b) $|z_1z_2| =$, $\arg(z_1z_2) = \pi, z_1z_2 = \cos\pi + i\sin\pi$.

(c) $|z_1z_2| = 100, \arg(z_1z_2) = 0, z_1z_2 = 100[\cos 0 + i\sin 0]$,

Complex Numbers
Polar Form

3. (a) (i) $|z_1| = 2$, $|z_2| = 3$, $\left|\frac{z_1}{z_2}\right| = \frac{2}{3}$

(ii) $\arg(z_1) = \frac{\pi}{2}$, $\arg(z_2) = \frac{\pi}{4}$, $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{4}$ (iii) $\frac{2}{3}[\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)]$

(b) (i) $|z_1| = \sqrt{2}$, $|z_2| = 2$, $\left|\frac{z_1}{z_2}\right| = \frac{\sqrt{2}}{2}$

(ii) $\arg(z_1) = \frac{\pi}{6}$, $\arg(z_2) = \frac{\pi}{3}$, $\arg\left(\frac{z_1}{z_2}\right) = \left(-\frac{\pi}{6}\right)$ (iii) $\frac{\sqrt{2}}{2}[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)]$

(c) (i) $|z_1| = r_1$, $|z_2| = r_2$, $\left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2}$

(ii) $\arg(z_1) = \theta_1$, $\arg(z_2) = \theta_2$, $\arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2$

(iii) $r_1 r_2[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

4. (a) $\frac{3}{2}$, $-\frac{\pi}{2}$, $\frac{3}{2}[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)]$.

(b) 4, $\frac{11\pi}{15}$, $4[\cos\left(\frac{11\pi}{15}\right) + i\sin\left(\frac{11\pi}{15}\right)]$.

(b) 1, $\frac{8\pi}{9}$, $\cos\left(\frac{8\pi}{9}\right) + i\sin\left(\frac{8\pi}{9}\right)$.

5. (a) $\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$ (b) $8[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)]$

(c) $20[\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)]$ (d) $\frac{1}{3}[\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)]$

Complex Numbers
De Moivre's Theorem

Questions

1. Simplify :-

(a) $[\cos\left(\frac{5\pi}{24}\right) + i\sin\left(\frac{5\pi}{24}\right)]^4$ (b) $[\cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)]^5$

(c) $\dfrac{(\cos 2\theta + i\sin 2\theta)^5}{(\cos 3\theta + i\sin 3\theta)^3}$ (d) $(\cos\theta + i\sin\theta)^8(\cos\theta - i\sin\theta)^4$ (note)

2. Since $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$, deduce expressions for $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$.

3. If $z = \cos\theta + i\sin\theta$, show that :-

(i) $z + \frac{1}{z} = 2\cos\theta$ and (ii) $z^2 + \frac{1}{z^2} = 2\cos 2\theta$

4. If $z = \cos\theta + i\sin\theta$, then :-

$z^n = (\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$ and

$z^{-n} = (\cos\theta + i\sin\theta)^{-n} = \cos(-n\theta) + i\sin(-n\theta)$.

Show that (i) $\cos(n\theta) = \frac{1}{2}\left(z^n + z^{-n}\right)$ and (ii) $\sin(n\theta) = \frac{1}{2i}\left(z^n - z^{-n}\right)$

Answers

1. (a) $\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

(b) $\cos 2\pi + i\sin 2\pi = 1$

(c) $\cos\theta + i\sin\theta$

(d) $\cos 4\theta + i\sin 4\theta$

2. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

3. Proofs

4. Proofs

Complex Numbers
De Moivre's Theorem

QUESTIONS

By considering expansions of $(\cos\theta + i\sin\theta)^2$ express

(a) $\cos 2\theta$ in terms of $\cos\theta$ and $\sin\theta$

(b) $\sin 2\theta$ in terms of $\cos\theta$ and $\sin\theta$

(c) $\tan 2\theta$ in terms of $\tan\theta$

ANSWERS

(a) $\cos^2\theta - \sin^2\theta$

(b) $2\sin\theta\cos\theta$

(c) $\dfrac{2\tan\theta}{1-\tan^2\theta}$

Complex Numbers

De Moivre's Theorem

QUESTION1

Using $2\cos(n\theta) = z^n + \frac{1}{z^n}$ and $2i\sin(n\theta) = z^n - \frac{1}{z^n}$

Show that

(a) $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$

(b) $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

(c) $\cos^3\theta = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta$

(d) $\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$

(e) $\sin^2 8\theta = \frac{1}{2} - \frac{1}{2}\cos 16\theta$

ANSWER1

(a) Proof (b) Proof (c) Proof (d) Proof

(e) Proof

Complex Numbers
Roots

Questions:

1. Work out $\sqrt{-4-i}$

2. Work out $\sqrt[3]{3+2i}$

3. Work out $\sqrt[5]{1}$

4. Work out $\sqrt[3]{-1}$

5. Find the square root of

   (a) $3-4i$

   (b) $21-20i$

   (c) $2i$

   (d) $-24+10i$

Answers:

1. $0.2481-2.0153i$ $\quad -0.2481+2.0153i$

2. $1.5040+0.2986i$ $\quad -1.0106+1.1532i$ $\quad -0.4934-1.4519i$

3. 1 $\quad 0.3090+0.9511i$ $\quad -0.8090+0.5878i$ $\quad -0.8090-0.5878i$ $\quad 0.3090-0.9511i$

4. -1 $\quad \frac{1}{2}+\frac{\sqrt{3}}{2}i$ $\quad \frac{1}{2}-\frac{\sqrt{3}}{2}i$

5. (a) $-2+i$ $\quad 2-i$ (b) $5-2i$ $\quad -5+2i$

   (c) $1+i$ $\quad -1-i$ (d) $1+5i$ $\quad -1-5i$

Complex Numbers
Fundamental Theorem of Algebra

QUESTION

1. What would you expect the roots of the following equations to be in terms of i ?

(a) $x^2 + 4 = 0$ (b) $x^2 + 9 = 0$ (c) $x^2 + 3 = 0$

2. Assuming that the normal formula for solving a quadratic equation can be used, solve the following quadratic equations:-

(a) $x^2 - 2x + 2 = 0$ (b) $x^2 - 4x + 5 = 0$

(c) $x^2 - 4x + 13 = 0$ (d) $x^2 + 2x + 2 = 0$

(e) $4x^2 - 4x + 5 = 0$ (f) $x^2 + 6x + 10 = 0$

(g) $2x^2 - 2x + 1 = 0$ (h) $9x^2 - 6x + 2 = 0$

3. Solve the following equations as fully as you can:-

(a) $x^3 - 1 = 0$ (b) $x^4 - 1 = 0$

(c) $x^3 - x^2 - x - 2 = 0$ (d) $(x^2 + 4)(x^2 + 9) = 0$

4. Simplify $(x - 1 - i)(x - 1 + i)$.

Hence state an equation which has $(1 + i)$ and $(1 - i)$ as its roots.

ANSWER

1. (a) $\pm 2i$ (b) $\pm 3i$ (c) $\pm \sqrt{3}i$

2. (a) $1 \pm i$ (b) $2 \pm i$ (c) $2 \pm 3i$ (d) $-1 \pm i$

(e) $\frac{1}{2} \pm i$ (f) $-3 \pm i$ (g) $\frac{1}{2} \pm \frac{1}{2}i$ (h) $\frac{1}{3} \pm \frac{1}{3}i$

3. (a) $x = 1$, $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ (b) $x = \pm 1, \pm i$ (c) $x = 2, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ (d) $x = \pm 2i, \pm 3i$

4. $x^2 - 2x + 2$ and $x^2 - 2x + 2 = 0$

Complex Numbers
Fundamental Theorem of Algebra

QUESTIONS

1. Find all the roots of the equation $z^3 - 11z + 20 = 0$.

2. Verify that $z = 1 + i$ is a root of the equation $z^4 + 3z^2 - 6z + 10 = 0$.
Hence find all the roots of the equation.

3. Verify that $z = -2 + 3i$ is a root of the equation $z^4 + 7z^2 - 12z + 130 = 0$.
Hence find all the roots of the equation.

4. Given that $2 - i$ is root of the equation $3z^3 - 10z^2 + 7z + 10 = 0$,
find the other roots.

5. Given that $1 - 2i$ is root of the equation $z^3 + z + 10 = 0$,
find the other roots.

6. Given that $3 + i$ is root of the equation $z^3 - 3z^2 - 8z + 30 = 0$,
find the other roots.

7. Show that $-1 + i$ is root of the equation $z^4 - 2z^3 - z^2 + 2z + 10 = 0$,
find the other roots.

ANSWERS

1.	$-4,\ 2 \pm i$	2.	$1 \pm i,\ -1 \pm 2i$	3.	$-2 \pm 3i,\ 2 \pm \sqrt{6}i$
4.	$-\frac{2}{3},\ 2 \pm i$	5.	$-2,\ 1 \pm 2i$	6.	$-3,\ 3 \pm i$
7.	$-1 \pm i,\ 2 \pm i$				