Binomial Theorem
General Practice

QUESTION

1. Use your calculator to find

(a) 7! (b) 4! (c) 0! (d) $\frac{12!}{5!}$ (e) $\frac{10!}{4!\,6!}$ (f) $\frac{12!+11!}{8!\,3!}$

2. Calculate

(a) $\binom{6}{4}$ (b) $\binom{10}{2}$ (c) $\binom{3}{0}$

3. Solve the following equations

(a) $\binom{n}{2}=6$ (b) $\binom{n}{2}=120$ (c) $\binom{2n}{2}=15$ (d) $\binom{n}{3}=35$

4. Expand and simplify as far as possible

(a) $(x+y)^3$ (b) $(x+1)^4$ (c) $(x-1)^5$ (d) $(x+2)^4$ (e) $(2x+1)^3$

(f) $(2x+4)^3$ (g) $(4a+3b)^5$ (h) $\left(\frac{x}{2}-3y\right)^4$ (i) $\left(\frac{3}{x}-2y\right)^3$

5. Find the coefficient of x^2 in the expansion of $(2x+3)^4$

6. Expand and simplify $(4x-5y)^4$. When $y=\frac{1}{x}$ find the term independent of x.

7. Expand $(1+x+y)^3$ by writing it as $[(1+x)+y]^3$.

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ANSWER1

1. (a) 5040 (b) 24 (c) 1 (d) 3991680 (e) 210 (f) 2145

2. (a) 15 (b) 45 (c) 1

3. (a) $n = 4$ (b) $n = 16$ (c) $n = 3$

   (d) solve $n^3 - 3n^2 + 2n - 210 = 0$ to get $n = 7$

   N.B. In Q3 ignore the negative answers.

4. (a) $x^3 + 3x^2y + 3xy^2 + y^3$ (b) $x^4 + 4x^3 + 6x^2 + 4x + 1$

   (c) $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$ (d) $x^4 + 8x^3 + 24x^2 + 32x + 16$

   (e) $8x^3 + 12x^2 + 6x + 1$ (f) $8x^3 + 48x^2 + 96x + 64$

   (g) $1024a^5 + 3840a^4b + 5760a^3b^2 + 4320a^2b^3 + 1620ab^4 + 243b^5$

   (h) $\frac{x^4}{16} - \frac{3x^3y}{2} + \frac{27x^2y^2}{2} - 54xy^3 + 81y^4$

   (i) $\frac{27}{x^3} - \frac{54y}{x^2} + \frac{36y^2}{x} - 8y^3$

5. Expansion is $16x^4 + 96x^3 + 216x^2 + 216x + 81$. So x^2 coefficient is 216.

6. Expansion is $256x^4 - 1280x^3y + 2400x^2y^2 - 2000xy^3 + 625y^4$. When $y = \frac{1}{x}$ the constant term is 2400.

7. Expansion is $1 + 3x + 3y + 3x^2 + 6xy + 3y^2 + x^3 + 3x^2y + 3xy^2 + y^3$.

Binomial Theorem
General Practice

QUESTION

1. By writing $x-y$ as $x[1-\frac{y}{x}]$, find an expression for $(x-y)^5$

2. Write down expressions for

(a) $(x+1)^5$ (b) $(1-x)^4$ (c) $(a+2b)^3$ (d) $(2a-b)^5$

(e) $(2x-3y)^5$ (f) $(x+\frac{1}{x})^5$ (g) $(a-2b)^6$ (h) $(2a+b)^7$

ANSWER

1. $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

2. (a) $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$ (b) $1 - 4x + 6x^2 - 4x^3 + x^4$
(c) $a^3 + 6a^2b + 12ab^2 + 8b^3$ (d) $32a^5 - 80a^4b + 80a^3b^2 - 40a^2b^3 + 10ab^4 - b^5$
(e) $32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5$
(f) $x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}$
(g) $a^6 - 12a^5b + 60a^4b^2 - 160a^3b^3 + 240a^2b^4 - 192ab^5 + 64b^6$
(h) $128a^7 + 448a^6b + 672a^5b^2 + 560a^4b^3 + 280a^3b^4 + 84a^2b^5 + 14ab^6 + b^7$

QUESTION

1. Expand the following using the Binomial Expansion :-

(a) $(3+x)^3$ (b) $(5+2x)^3$ (c) $(3+x)^4$

(d) $(2-x)^4$ (e) $(x+2y)^3$ (f) $(2x-3y)^3$

(g) $(1+3x)^4$ (h) $(2-3x)^5$ (i) $(2x+\frac{3}{x})^5$

2. Expand $(2+x)^5$ and use your expansion to find (a) $(2{\cdot}1)^5$ (b) $(1{\cdot}9)^5$

3. Expand $(2+x)^7$ in ascending powers of x up to and including the term in x^3. Hence evaluate $(2{\cdot}1)^7$ correct to 6 significant figures.

4. Use the Binomial Theorem to evaluate the following to the stated degree of accuracy.

(a) $(1{\cdot}01)^4$ correct to 5 decimal places.

(b) $(0{\cdot}998)^5$ correct to 7 significant figures

(c) $(0{\cdot}99)^{10}$ correct to 4 decimal places.

(d) $(1{\cdot}99)^{10}$ correct to 4 significant figures.

Binomial Theorem
General Practice

ANSWERS

1. (a) $27 + 27x + 9x^2 + x^3$ (b) $125 + 150x + 60x^2 + 8x^3$
   (c) $81 + 108x + 54x^2 + 12x^3 + x^4$ (d) $16 - 32x + 24x^2 - 8x^3 + x^4$
   (e) $x^3 + 6x^2y + 12xy^2 + 8y^3$ (f) $8x^3 - 36x^2y + 54xy^2 - 27y^3$
   (g) $1 + 12x + 54x^2 + 108x^3 + 81x^4$
   (h) $32 - 240x + 720x^2 - 1080x^3 + 810x^4 - 243x^5$
   (i) $32x^5 + 240x^3 + 720x + \frac{1080}{x} + \frac{810}{x^3} + \frac{243}{x^5}$

2. (a) 40·84101 (b) 24·76099

3. 180·109

4. (a) 1·04060 (b) 0·9900399 (c) 0·9044 (d) 973·9

Binomial Theorem
General Term

Questions:

1. Write down and simplify the general term in the expansion of $\left(x^2+\frac{1}{x}\right)^{10}$.
   Hence, or otherwise, obtain the term in x^{14}.

2. Write down and simplify the general term in the expansion of $\left(2x-\frac{1}{x^2}\right)^{9}$.
   Hence, or otherwise, obtain the term independent of x.

3. Write down and simplify the general term in the expression $\left(\frac{2}{x}+\frac{1}{4x^2}\right)^{10}$.

   Hence, or otherwise, obtain the term in $\frac{1}{x^{13}}$.

4. State and simplify the general term in the binomial expansion of $\left(2x-\frac{5}{x^2}\right)^{6}$.
   Hence, or otherwise, find the term independent of x.

5. Find the coefficient of x^2y^4 in the expansion of $(x + 2y)^6$

6. Find the coefficient of x^9 in the expansion of $(1 + 3x^3)^4$

7. Find the coefficient of x^2 in the expansion of $\left(x + \frac{1}{x^2}\right)^5$

8. Find the coefficient of x^4y^3 in the expansion of $(x - y)^7$

9. Find the coefficient of x^2y^2 in the expansion of $(2x - y)^4$

10. Find the coefficient of x^6 in the expansion of $(1 + x^2)^8$

11. Find the coefficient of y^5 in the expansion of $\left(y - \frac{1}{y}\right)^5$

Answers:

1.	45	2.	–5376	3.	240	4.	6000
5.	240	6.	108	7.	5	8.	–35
9.	24	10.	56	11.	1		