ADVANCED HIGHER MATHEMATICS

Exam Questions on Systems of Linear Equations

1.   Use Gaussian elimination to solve the following system of equations

$$x + y + \ z = 10$$
$$2x - y + 3z = \ 4$$
$$x \quad + 2z = 20.$$

2.   Use Gaussian elimination to solve the following system of equations

$$x + \ y + 3z = 2$$
$$2x + \ y + \ z = 2$$
$$3x + 2y + 5z = 5.$$

3.   Use Gaussian elimination to solve the system of equations

$$x + y = 4$$
$$x + 2y - z = 5$$
$$3y - 2z = 6.$$

4.   Use Gaussian elimination to solve the following system of equations.

$$x \quad\quad - \ z \ = \ 2$$
$$2y \ - \ 3z \ = \ 6$$
$$2x \ + \ y \ + \ z \ = \ 1$$

5.   Use Gaussian elimination to solve the system of equations

$$x + y + z = 0$$
$$2x - y + z = -1 \cdot 1$$
$$x + 3y + 2z = 0 \cdot 9$$

6.   Use Gaussian elimination to solve the following system of equations

$$x + y - z = 6$$
$$2x - 3y + 2z = 2$$
$$-5x + 2y - 4z = 1.$$

7.   Solve the system of equations

$$x + y - z = \ 3$$
$$y + z = -2$$
$$x - 3z = \ 7$$

8.   Solve the system of equations

$$x + y \quad = 1$$
$$2x + 3y + z = 2$$
$$2x + 2y + z = 1$$

9.   Below is a system of equations:

$$x + 2y + 3z = 3$$
$$2x - y + 4z = 5$$
$$x - 3y + 2\lambda z = 2$$

Use Gaussian elimination to find the value of λ which leads to redundancy.

10.   (a)(i)   Use Gaussian elimination on the system of equations below to give an expression for z in terms of λ.

$$x + 2y - z = -3$$
$$4x - 2y + 3z = 11$$
$$3x + y + 2\lambda z = 8$$

   (ii)   For what value of λ is this system of equations inconsistent?

   (b)   Determine the solution of this system when $\lambda = -2 \cdot 5$.

11.   Below is a system of equations:

$$x - 2y + z = -4$$
$$3x - 5y - 2z = 1$$
$$-7x + 11y + \lambda z = -11$$

Use Gaussian elimination to find the value of λ which leads to redundancy.

12.   (a)(i)   Use Gaussian elimination on the system of equations below to give an expression for z in terms of λ.

$$x + y + z = 2$$
$$4x + 3y - \lambda z = 4$$
$$5x + 6y + 8z = 11$$

   (ii)   For what value of λ is this system of equations inconsistent?

   (b)   Determine the solution of this system when $\lambda = 2$.

13. (a) Use Gaussian elimination to solve the system of equations below when $a = 3$.

$$x - y + z = 1$$
$$x + y + 2z = 0$$
$$2x - y + az = 2$$

(b) Explain what happens when $a = 2 \cdot 5$.

14. (a) Use Gaussian elimination to solve the system of equations below when $a = 6$:

$$x + y + 3z = 1$$
$$3x + ay + z = 1$$
$$x + y + z = -1.$$

(b) Explain what happens when $a = 3$.

15. (a) Use Gaussian elimination to solve the system of equations below when $\lambda \neq 2$:

$$x + y + 2z = 1$$
$$2x + \lambda y + z = 0$$
$$3x + 3y + 9z = 5.$$

(b) Explain what happens when $\lambda = 2$.

16. (a) Use Gaussian elimination to show that the system of equations below has an infinite number of possible solutions.

$$2x - y + 2z = 1$$
$$x + y - 2z = 2$$
$$x - 2y + 4z = -1.$$

(b) If $z = t$, obtain solutions for x and y in terms of t.

17. (a) Use Gaussian elimination to obtain the solution of the following system of equations in terms of the parameter λ.

$$4x + 6z = 1$$
$$2x - 2y + 4z = -1$$
$$-x + y + \lambda z = 2$$

(b) Describe what happens when $\lambda = -2$.

(c) When $\lambda = -1 \cdot 9$ the solution is $x = -22 \cdot 25$, $y = 8 \cdot 25$, $z = 15$.

Find the solution when $\lambda = -2 \cdot 1$.
Comment on these solutions.