ADVANCED HIGHER MATHEMATICS

Exam Questions on Sequences and Series

1. The first term of an arithmetic sequence is 2 and the 20th term is 97.
Obtain the sum of the first 50 terms.

2. The sum of the first twenty terms of an arithmetic series is 320.
The twenty-first term is 37.
What is the sum of the first ten terms?

3. (a) $u_1, u_2, \ldots, u_n, \ldots$ is an arithmetic sequence. The first term, u_1, is equal to 45 and the third term, u_3, is equal to 5.
Find u_{11}.

(b) $v_1, v_2, \ldots, v_n, \ldots$ is an geometric sequence of **positive** numbers.
The first term, v_1, is equal to 45 and the third term, v_3, is equal to 5.
Calculate the sum to infinity of the associated geometric series.

4. The second and third terms of a geometric sequence are –6 and 3 respectively.
Explain why the sequence has a sum to infinity, and obtain this sum.

5. A geometric sequence has second and fifth terms 108 and 4 respectively.

(a) Calculate the value of the common ratio.

(b) State why the associated geometric series has a sum to infinity.

(c) Find the value of this sum to infinity.

6. The fifth term of an arithmetic sequence is −6 and the twelfth term is −34.

(a) Determine the values of the first term and the common difference.

(b) Obtain algebraically the value of *n* for which $S_n = -144$.

7. The sum, S_n, of the first *n* terms of a sequence $u_1, u_2, u_3, \ldots$ is given by
$S_n = 8n - n^2,\ n \geq 1$.

Calculate the values of u_1, u_2 and u_3 and state what type of sequence it is.

Obtain a formula for u_n in terms of *n*, simplifying your answer.

8. A geometric sequence has first term 80 and common ratio $\frac{1}{3}$.

(a) For this sequence, calculate:

(i) the 7th term;
(ii) the sum to infinity of the associated geometric series.

The first term of this geometric sequence is equal to the first term of an arithmetic sequence. The sum of the first five terms of this arithmetic sequence is 240.

(b) (i) Find the common difference of this sequence.
(iii) Write down and simplify an expression for the *n*th term.

Let S_n represent the sum of the first *n* terms of this arithmetic sequence.

(c) Find the values of *n* for which $S_n = 144$.

9. The first and fourth terms of a geometric series are 2048 and 256 respectively. Calculate the value of the common ratio.

Given the sum of the first *n* terms is 4088, find the value of *n*.

10. The first three terms of an arithmetic sequence are *a*, $\frac{1}{a}$, 1 where $a < 0$.

Obtain the value of *a* and the common difference.

Obtain the smallest value of *n* for which the sum of the first *n* terms is greater than 1000.

11. Three terms of an arithmetic sequence, u_3, u_7 and u_{16}, form the first three terms of a geometric sequence.

Show that $a = \frac{6}{5}d$, where *a* and *d* are, respectively, the first term and common difference of the arithmetic sequence with $d \neq 0$.

Hence find the value of *r*, the common ratio of the geometric sequence.

12. Let $S_n = \sum_{r=1}^{n}(11-2r)$, $n \geq 1$.

(a) Obtain an expression for S_n in terms of *n*.

(b) Find the values of *n* for which $S_n = 21$.

13. (a) Show that $\sum_{r=1}^{n}(6r^2 - r) = \frac{1}{2}n(n+1)(4n+1)$.

(b) Hence evaluate $\sum_{r=5}^{10}(6r^2 - r)$.

14. (a) Show that $\sum_{r=1}^{n}(4-6r) = n-3n^2$.

(b) Hence write down a formula for $\sum_{r=1}^{2q}(4-6r)$.

(c) Show that $\sum_{r=q+1}^{2q}(4-6r) = q-9q^2$.

15. (a) Find the value of N for which $\sum_{r=1}^{N} r = 210$.

(b) Evaluate $\sum_{r=1}^{N} r^2$ for this value of N.

16. (a) Show that $\sum_{r=1}^{n}(r^3-3r) = \dfrac{n(n+1)(n-2)(n+3)}{4}$.

(b) Use the above result to evaluate $\sum_{r=5}^{15}(r^3-3r)$.

17. Obtain an expression for $\sum_{r=1}^{n}(2r^3+r^2-r)$ in terms of n.

Express your answer as a single algebraic fraction in its fully factorised form.

18. S_n is defined by $\sum_{r=1}^{n}\left(r^2+\dfrac{1}{3}r\right)$.

(a) Find an expression for S_n, fully factorising your answer.

(b) Hence find an expression for $\sum_{r=10}^{2p}\left(r^2+\dfrac{1}{3}r\right)$ where $p > 5$.