ADVANCED HIGHER MATHEMATICS

Exam Questions on Proof by Induction

1. Prove by induction that, for all integers $n \geq 1$,

$$\sum_{r=1}^{n}(3r-1) = \frac{1}{2}n(3n+1).$$

2. Prove by induction that

$$\sum_{r=1}^{n} r(3r-1) = n^2(n+1), \qquad \forall n \in \mathbf{N}.$$

3. Prove by induction that, for all positive integers *n*,

$$\sum_{r=1}^{n} 3^{r-1} = \frac{1}{2}(3^n - 1).$$

4. (a) Prove by induction that, for all natural numbers $n \geq 1$,

$$\sum_{r=1}^{n} 3(r^2 - r) = (n-1)n(n+1).$$

(b) Hence evaluate $\sum_{r=11}^{40} 3(r^2 - r)$.

5. Prove by induction that, for all positive integers *n*,

$$\sum_{r=1}^{n}(4r^3 + 3r^2 + r) = n(n+1)^3.$$

6. Prove by induction that, for all positive integers *n*,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}.$$

7. Prove by induction that, for all positive integers *n*,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}.$$

8. Prove by induction that, for all positive integers *n*,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}.$$