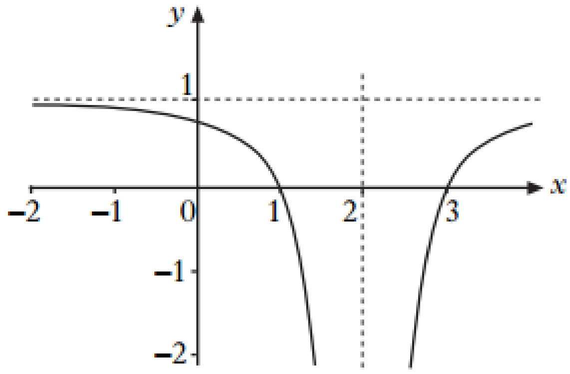
ADVANCED HIGHER MATHEMATICS

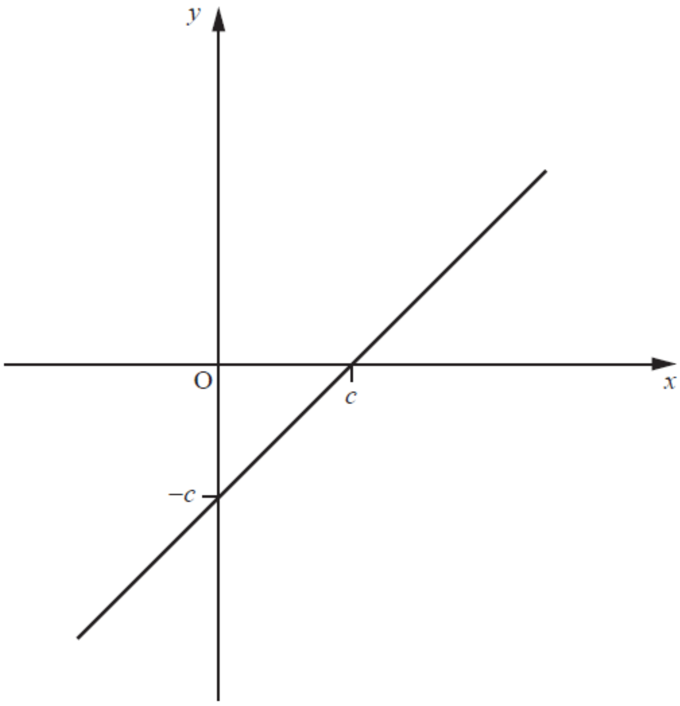
Exam Questions on Functions and Graphs

1. Part of the graph of $y = f(x)$ is shown below, where the dotted lines indicate asymptotes.



Sketch the graph of $y = -f(x+1)$, showing its asymptotes.
Write down the equations of the asymptotes.

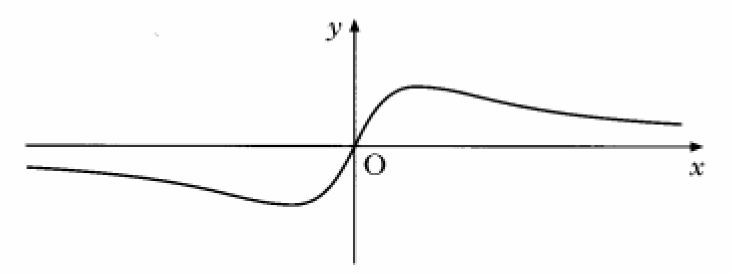
2. Below is a diagram showing the graph of a linear function, $y = f(x)$.



On separate diagrams, show the graph of:

(a) $y = |f(x) - c|$ (b) $y = |2f(x)|$

3.

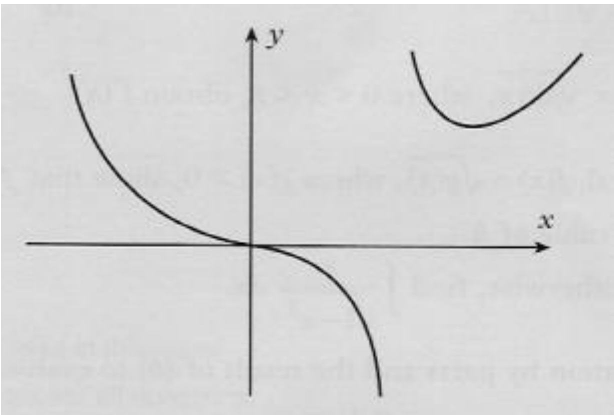


The diagram shows the shape of the graph of $y = \dfrac{x}{1+x^2}$.

(a) Obtain the stationary points of the graph.

(b) Sketch the graph of $y = \left|\dfrac{x}{1+x^2}\right|$ and identify its three critical points.

4. The diagram shows part of the graph of $y = \dfrac{x^3}{x-2}$, $x \neq 2$.



(a) Write down the equation of the vertical asymptote.

(b) Find the coordinates of the stationary points of the graph of $y = \dfrac{x^3}{x-2}$.

(c) Write down the coordinates of the stationary points of the graph of $y = \left|\dfrac{x^3}{x-2}\right| + 1$.

5. A function f is defined by $f(x) = \dfrac{x^2 + 6x + 12}{x + 2}$, $x \neq -2$.

(a) Express $f(x)$ in the form $ax + b + \dfrac{b}{x+2}$, stating the values of a and b.

(b) Write down an equation for each of the two asymptotes.

(c) Show that $f(x)$ has two stationary points.
Determine the coordinates and nature of the stationary points.

(d) Sketch the graph of f.

(e) State the range of values of k such that the equation $f(x) = k$ has no real solutions.

6. Consider the function $f(x) = \dfrac{x^2 + 3x + 6}{x + 2}$, $x \in \mathbf{R}$, $x \neq -2$.

(a)(i) Write down the equation of the vertical asymptote.
(ii) Show that $y = f(x)$ has a non-vertical asymptote and obtain its equation.

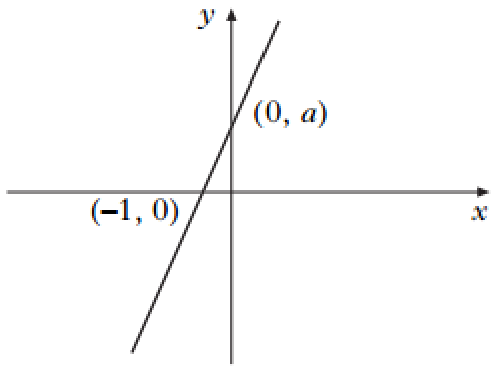
(b) Find the coordinates of the stationary points of $f(x)$ and justify their nature.

7. A function is defined by $f(x) = |x + 2|$ for all x.

(a) Sketch the graph of the function for $-3 \leq x \leq 3$.

(b) On a separate diagram, sketch the graph of $f'(x)$.

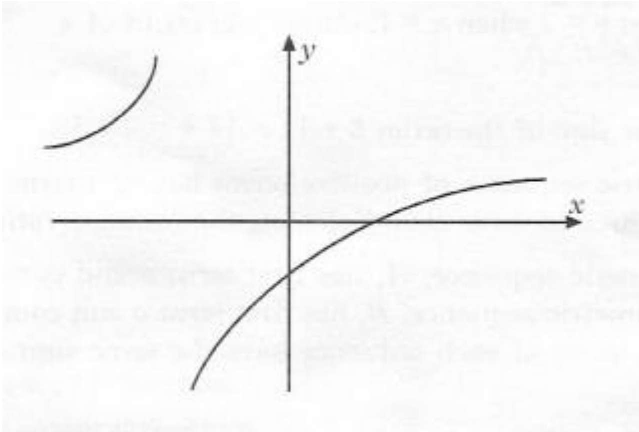
8.



The diagram shows part of the graph of a function $f(x)$.

Sketch the graph of $y = |f^{-1}(x)|$ showing the points of intersection with the coordinate axes.

9. The function *f* is defined by $f(x)=\dfrac{x-3}{x+2}$, $x\neq -2$, and the diagram shows part of its graph.



(a) Obtain algebraically the asymptotes of the graph of *f*.

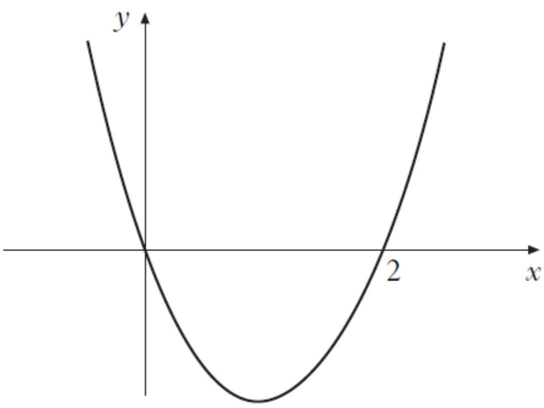
(b) Prove that the graph of *f* has no stationary points and no points of inflexion.

10. Determine whether $f(x)=x^2\sin x$ is odd, even or neither. Justify your answer.

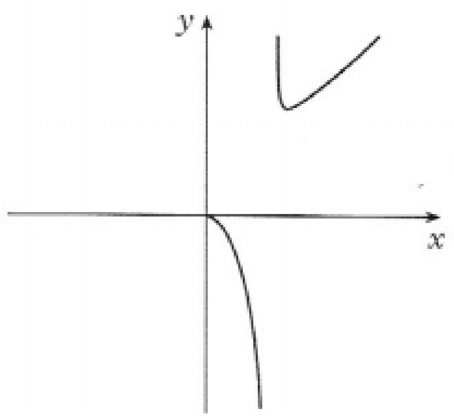
11. Determine whether the function $f(x)=x^4\sin 2x$ is odd, even or neither.
Justify your answer.

12. The diagram below shows part of the graph of a function $f(x)$.

State whether $f(x)$ is odd, even or neither.
Fully justify your answer.



13.

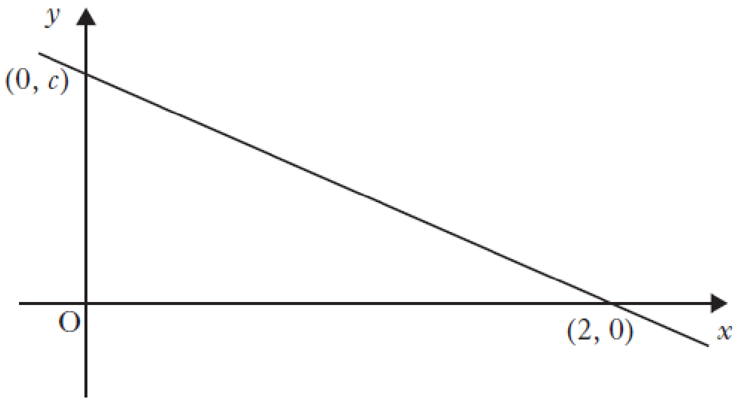


The diagram shows part of the graph of a function *f* which satisfies the following conditions:

(i) *f* is an even function;
(ii) two of the asymptotes of the graph $y = f(x)$ are $y = x$ and $x = 1$.

Copy the diagram and complete the graph. Write down equations for the other two asymptotes.

14. Part of the straight line graph of a function $f(x)$ is shown.



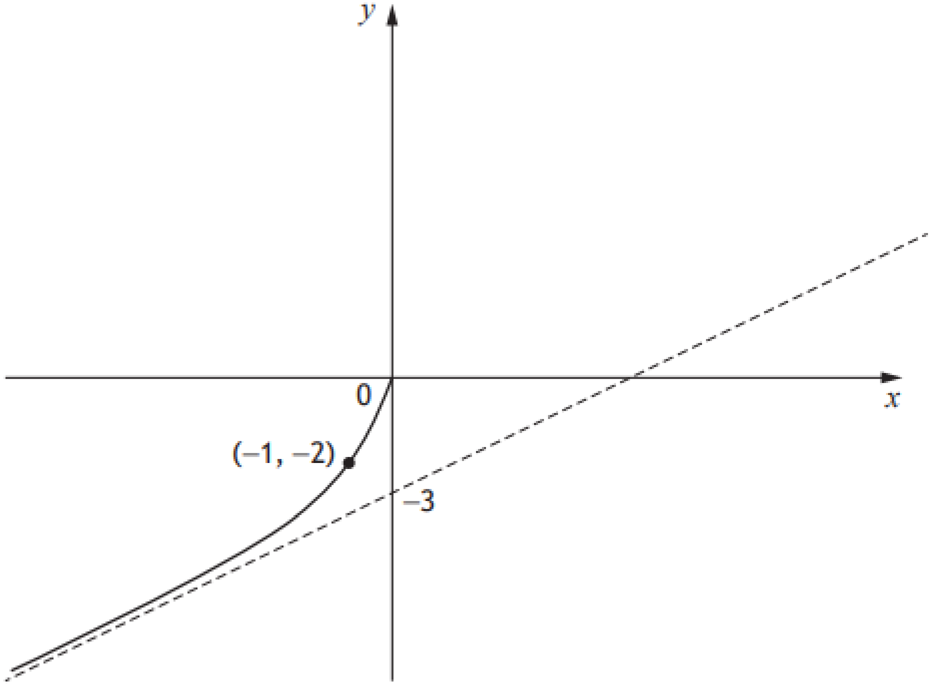
(a) Sketch the graph of $f^{-1}(x)$, showing points of intersection with the axes.

(b) State the value of k for which $f(x) + k$ is an odd function.

(c) Find the value of h for which $|f(x+h)|$ is an even function.

15. In the diagram below part of the graph of $y = f(x)$ has been omitted.

The point (–1, –2) lies on the graph and the line $y = \frac{1}{2}x - 3$ is an asymptote.



Given that $f(x)$ is an odd function:

(a) Copy and complete the diagram, including any asymptotes and any points you know to be on the graph.

(b) $g(x) = |f(x)|$. On a separate diagram, sketch $g(x)$.
Include any known asymptotes and points.

(c) State the range of values of $f'(x)$ given that $f'(0) = 2$.

16. For some function *f*, define

$$g(x) = f(x) + f(-x) \quad \text{and}$$
$$h(x) = f(x) - f(-x).$$

(a) Show that $g(x)$ is an even function and that $h(x)$ is an odd function.

(b) Hence show that $f(x)$ can be expressed as the sum of an even and an odd function.