ADVANCED HIGHER MATHEMATICS

Exam Questions on Complex Numbers 1

1. Given that $z_1 = 2 + 3i$ and $z_2 = p - 6i$, $p \in \mathbf{R}$, find:

   (a) $z_1 \bar{z}_2$

   (b) the value of p such that $z_1 \bar{z}_2$ is a real number.

2. Given the equation $z + 2i\bar{z} = 8 + 7i$, where $\bar{z}$ is the complex conjugate of z, express z in the form $a + bi$.

3. Given that $z = -1 + 2i$ is a root of the equation

   $$z^3 + 5z^2 + 11z + 15 = 0,$$

   obtain all the roots.

4. Verify that $z = 1 + 2i$ is a root of the equation

   $$z^3 + 3z^2 - 5z + 25 = 0.$$

   Obtain the other roots of this equation.

5. Verify that $z = 1 + i$ is a root of the equation

   $$z^3 + 16z^2 - 34z + 36 = 0.$$

   Obtain the other roots of this equation.

6. Show that $z = 3 + i$ is a root of the equation $z^3 - 8z^2 + 22z - 20 = 0$ and obtain the other roots of this equation.

7. Show that $z = 3 + 3i$ is a root of the equation $z^3 - 18z + 108 = 0$ and obtain the other roots of the equation.

8. Verify that $z = i$ is a root of the equation $z^4 + 4z^3 + 3z^2 + 4z + 2 = 0$.
   Hence find all the roots of this equation.

9. Given that $z = -2 + i$ is one root of the equation

   $$z^4 + z^3 - z^2 + 9z + 30 = 0,$$

   find all the other roots of this equation.

10. The complex number $z = 2 + i$ is a root of the polynomial equation

$$z^4 - 6z^3 + 16z^2 - 22z + q = 0, \quad \text{where } q \in \mathbf{Z}.$$

(a) State a second root of the equation.

(b) Find the value of q and the remaining roots.