ADVANCED HIGHER MATHEMATICS

Exam Questions on the Binomial Theorem

1. Obtain the binomial expansion of $(a^2 - 3)^4$.

2. Use the binomial theorem to expand $\left(\dfrac{1}{2}x - 3\right)^4$ and simplify your answer.

3. Express the binomial expansion of $\left(x - \dfrac{2}{x}\right)^4$ in the form $ax^4 + bx^2 + c + \dfrac{d}{x^2} + \dfrac{e}{x^4}$ for integers a, b, c, d and e.

4. Obtain the binomial expansion of $\left(b - \dfrac{2}{b}\right)^5$ and simplify the expression.

5. Expand
$$\left(x^2 - \dfrac{2}{x}\right)^4, \qquad x \neq 0,$$
and simplify as far as possible.

6. Use the binomial theorem to expand $\left(x^3 - \dfrac{2}{x}\right)^4$ and simplify your answer.

7. Expand and simplify $\left(2a - \dfrac{3}{a}\right)^4$.

8. Write down the binomial expansion of $\left(3x - \dfrac{2}{x^2}\right)^4$ and simplify your answer.

9. Write down the binomial expansion of $\left(\dfrac{2}{y^2} - 5y\right)^3$ and simplify your answer.

10. Use the binomial theorem to expand and simplify $\left(\dfrac{x^2}{3} - \dfrac{2}{x}\right)^5$.

11. (a) Write down and simplify the binomial expansion of $(e^x + 2)^4$.

    (b) Hence obtain $\int (e^x + 2)^4 dx$.

12. Write down and simplify the general term in the expansion of $(x^2 + 3x)^8$.

Hence, or otherwise, obtain the coefficient of x^{13}.

13. Write down and simplify the general term in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^{10}$.

Hence, or otherwise, obtain the term in x^{14}.

14. (a) Write down and simplify the general term in the binomial expansion of $\left(2x + \dfrac{5}{x^2}\right)^9$.

(b) Hence, or otherwise, find the term independent of x.

15. Write down and simplify the general term in the binomial expansion of $\left(\dfrac{1}{x} + 3x\right)^{10}$.

Hence, or otherwise, find the term in x^6.

16. Write down and simplify the general term in the binomial expansion of $\left(\dfrac{3}{x} - 2x\right)^{13}$.

Hence, or otherwise, find the term in x^9.

17. Write down and simplify the general term in the expansion of $\left(2x - \dfrac{1}{x^2}\right)^9$.

Hence, or otherwise, obtain the term independent of x.

18. State and simplify the general term in the binomial expansion of $\left(2x - \dfrac{5}{x^2}\right)^6$.

Hence, or otherwise, find the term independent of x.

19. Write down and simplify the general term in the expansion of $\left(\dfrac{2}{x} + \dfrac{1}{4x^2}\right)^{10}$.

Hence, or otherwise, obtain the term in $\dfrac{1}{x^{13}}$.

20. Show that
$$\binom{n+2}{3} - \binom{n}{3} = n^2,$$

for all integers n, where $n \geq 3$.

[*Note*: $\binom{n}{r}$ denotes the binomial coefficient ^nC_r in this question.]

21. Show that

$$\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$$

where the integer n is greater than or equal to 3.

[*Note*: $\binom{n}{r}$ denotes the binomial coefficient ^nC_r in this question.]