EXEMPLAR PAPER 2

1a) $u_1 = \frac{-1}{2} \times -16 = 8$

$u_2 = -\frac{1}{2} \times 8 = -4$

b) $V_2 = pV_1 + q \quad V_3 = pV_2 + q$

$5 = 4p + q$ ① $\quad 7 = 5p + q$ ②

② − ① $\quad 2 = p$

$q = -3$

c) i) sequence in (a) limit $= \frac{b}{1-a} = \frac{0}{1-\frac{1}{2}} = 0$

ii) The multiplier needs to be between −1 and 1. 2 isn't.

2a) $2x - y = -5$

$y = 2x + 5$

$x^2 + (2x+5)^2 - 6x - 2(2x+5) - 30 = 0$

$x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$

$5x^2 + 10x - 15 = 0$

$5(x^2 + 2x - 3) = 0$

$5(x+3)(x-1)$

$x = -3 \quad x = 1$

$y = -1 \quad y = 7$

$(-3, -1) \quad (1, 7)$

b) centre circle 1 $(3, 1)$

midpoint PQ $(-1, 3)$

centre circle 2 $(-5, 5)$

radius circle 1 $= \sqrt{(-3)^2 + (-1)^2 + 30}$

$= \sqrt{40}$

circle 2 $(x+5)^2 + (y-5)^2 = 40$

3. $b^2 - 4ac < 0$

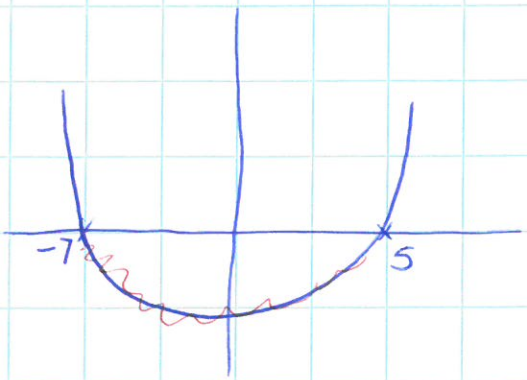
$(p+1)^2 - 4 \times 1 \times 9 < 0$

$p^2 + 2p + 1 - 36 < 0$

$p^2 + 2p - 35 < 0$

$(p + 7)(p - 5)$

$p = -7 \quad p = 5$



$-7 < p < 5$

4. $\int_{-3}^{0} x^3 + 3x^2 + 2x + 3 - (2x + 3)\ dx$

$\int_{-3}^{0} x^3 + 3x^2\ dx$

$= \left[\frac{x^4}{4} + x^3 \right]_{-3}^{0}$

$= 0 - \left(\frac{(-3)^4}{4} + (-3)^3 \right)$

$= 0 - \left(\frac{81}{4} - 27 \right)$

$= 0 - \frac{-27}{4}$

$= \frac{27}{4}$ units^2

5. a) $\overrightarrow{OB} = 4i + 4j + 0k$

b) $\overrightarrow{DB} = b - d$

$= \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$

$\overrightarrow{DM} = m - d$

$= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$

c) $\overrightarrow{DB} \cdot \overrightarrow{DM} = 0 - 4 + 36 = 32$

$|\overrightarrow{DB}| = \sqrt{4 + 4 + 36}$

$= \sqrt{44}$

$|\overrightarrow{DM}| = \sqrt{0 + 4 + 36}$

$= \sqrt{40}$

$\cos\theta = \frac{32}{\sqrt{44} \times \sqrt{40}} = 0.762\ldots$

$\theta = 40.3^\circ$

6. $p.(p+q+r) = p.p + p.q + p.r$

$= 3^2 + 3\times3\times\cos60 + 0$

$= 9 + \frac{9}{2}$

$= \frac{27}{2}$

7. a) $0.5 = 1\times e^{-k\times25}$

$0.5 = e^{-25k}$

$\ln0.5 = \ln e^{-25k}$

$\ln0.5 = -25k \ln e$

$k = \frac{\ln0.5}{-25} = 0.0277\ldots = 0.028$

b) Assume initial concentration = 100%

$P_t = 100e^{-0.028\times30}$

$= 10.6\%$ Statement incorrect. Reduced by 89.4%

8. $\int_{\frac{\pi}{8}}^{a} 5\sin\left(4x - \frac{\pi}{2}\right)dx$

$= \left[-\frac{5}{4}\cos\left(4x - \frac{\pi}{2}\right)\right]_{\frac{\pi}{8}}^{a}$

$= -\frac{5}{4}\cos\left(4a - \frac{\pi}{2}\right) + \frac{5}{4}\cos\left(\frac{4\pi}{8} - \frac{\pi}{2}\right)$

$= -\frac{5}{4}\cos\left(4a - \frac{\pi}{2}\right) + \frac{5}{4} = \frac{10}{4}$

$-\frac{5}{4}\cos\left(4a - \frac{\pi}{2}\right) = \frac{5}{4}$

$\cos\left(4a - \frac{\pi}{2}\right) = -1$

$4a - \frac{\pi}{2} = \pi$

$4a = \frac{3\pi}{2}$

$a = \frac{3\pi}{8}$

9. a) $L = 3x + 48x^{-1}$

$L' = 3 - 48x^{-2} = 0$

$\frac{48}{x^2} = 3$

$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$

$x = 4$

x	$\rightarrow 4 \rightarrow$
L'	$+ \; 0 \; -$
slope	/ — \

max when $x = 4$

b) $L = 3(4) + \frac{48}{4} = 24$ metres

$24 \times 8.25 = €198$ Incorrect. €3 more than claim

10. a) Acceleration $= v'(t) = 2 \times 8 \sin\left(2t - \frac{\pi}{2}\right) = a(t)$

b) $a(t) = -16 \sin\left(2 \times 10 - \frac{\pi}{2}\right) = +6.53$ * calc in radian *

positive ∴ increasing

~~positive ∴ increasing negative ∴ decreasing~~

c) $s(t) = \int v(t)\, dt$

$= \int 8 \cos\left(2t - \frac{\pi}{2}\right) dt$

$= \frac{8}{2} \sin\left(2t - \frac{\pi}{2}\right) + C$ sub in $(0, 4)$

$4 = 4 \sin\left(2 \times 0 - \frac{\pi}{2}\right) + C$

$4 = 4 \times -1 + C$

$C = 8$

$s(t) = 4 \sin\left(2t - \frac{\pi}{2}\right) + 8$