# 2015 Higher Maths

## Old Specification

### Paper 1 Section A

(1) $f(x) = 2x^3 - 7$
$f'(x) = 6x^2$
$f'(2) = 6(2^2)$
$= 24$

**Ans: C**

(2) $2y = 3x + 5$
$y = \frac{3}{2}x + \frac{5}{2}$

$m_1 = \frac{3}{2}$

$m_2 = -\frac{2}{3}$

**Ans: B**

(3)
$$\begin{array}{r|rrrr} 2 & 2 & 1 & -4 & 1 \\ & \downarrow & 4 & 10 & 12 \\ \hline & 2 & 5 & 6 & \boxed{13} \end{array}$$

**Ans: D**

(4) $y = -3\cos 2x$

**Ans: A**

(5) $u_5 = 0{\cdot}2u_4 + 9$
$11 = 0{\cdot}2u_4 + 9$
$2 = 0{\cdot}2u_4$
$u_4 = 10$

$u_4 = 0{\cdot}2u_3 + 9$
$10 = 0{\cdot}2u_3 + 9$
$1 = 0{\cdot}2u_3$
$u_3 = 5$

**Ans: C**

(6) $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$

Ratio 3 : 2

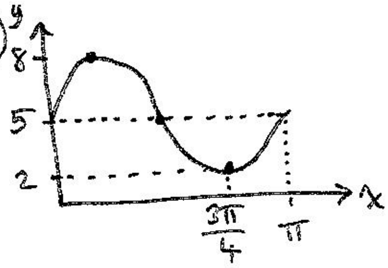
**Ans: B**

(7) $\int (x+4)(x-4)\,dx$
$= \int (x^2 - 16)\,dx$
$= \frac{x^3}{3} - 16x + c$

**Ans: C**

(8) $m = \tan 60°$
$= \sqrt{3}$

**Ans: D**

(9)



**Ans: B**

(10) $\cos x = -\frac{1}{2}$

Related acute angle $= \cos^{-1}\frac{1}{2}$
$= \frac{\pi}{3}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$

**Ans: D**

(11) $\int (4x-1)\,dx$

$y = 2x^2 - x + c$
$9 = 2(2^2) - 2 + c$
$9 = 8 - 2 + c$
$9 = 6 + c$
$c = 3$
$y = 2x^2 - x + 3$

**Ans: C**

(12) $\overrightarrow{RT} = \begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix}$

$x = 3 + 6 = 9$
$y = -1 + 3 = 2$
$z = 2 - 9 = -7$
$(9, 2, -7)$

**Ans: C**

(13) $a > 0$ because ∪
but $b^2 - 4ac < 0$
because no real roots.

**Ans: B**

(14) $\cos 2x = 2\cos^2 x - 1$
$= 2\left(-\frac{2}{5}\right)^2 - 1$
$= 2\left(\frac{4}{25}\right) - 1$
$= \frac{8}{25} - \frac{25}{25}$
$= -\frac{17}{25}$

**Ans: D**

(15) $y = k(x-3)(x+1)(x+2)$

Subs. $(0,-3)$:

$-3 = k(-3)(1)(2)$

$-3 = k(-6)$

$k = \frac{1}{2}$

Ans: A

(16) $e^{4t} = 6$

$\ln e^{4t} = \ln 6$

$4t = \ln 6$

$t = \frac{1}{4}\ln 6$

Ans: B

(17) $|\underline{u}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$

$= \sqrt{\frac{25}{25}}$

$= 1$

$-10\left(\frac{3}{5}\right) = -6$

$-10(1) = -10$

Ans: D

(18) $g = -6,\ f = -5$

Centre $(6,5)$

$r = 6$

$r = \sqrt{g^2+f^2-k}$

$6 = \sqrt{36+25-k}$

$k = 25$

Ans: C

(19) $\frac{1}{2} - 2\left(\frac{1}{2}\right)$

$= \frac{1}{2} - 1$

$= -\frac{1}{2}$

Ans: B

(20) $x \mapsto \frac{1}{2}x$

$y \mapsto -y$

So $(a,b) \mapsto \left(\frac{a}{2}, -b\right)$

Ans: A

## Section B

(21)(a)

| | | | | |
|---|---|---|---|---|
| 1 | 1 | -6 | 9 | -4 |
| | ↓ | 1 | -5 | 4 |
| | 1 | -5 | 4 | 0 |

Remainder = 0

so $(x-1)$ is a factor.

$(x-1)(x^2-5x+4)$

$= (x-1)(x-1)(x-4)$

$= (x-1)^2(x-4)$

(b)(i) $\frac{dy}{dx} = 3x^2 - 12x + 11$

$m = 3(1^2) - 12(1) + 11$

$= 3 - 12 + 11$

$= 2$

$y = mx + c$

$3 = 2(1) + c$

$c = 1$

$y = 2x + 1$

(ii) $x^3 - 6x^2 + 11x - 3 = 2x + 1$

$x^3 - 6x^2 + 9x - 4 = 0$

$(x-1)^2(x-4) = 0$

$x_B = 4$

$y_B = 2(4) + 1$

$= 9$

$B(4,9)$

(22) $f(x) = 4x^{-2} + x$

$f'(x) = -8x^{-3} + 1 = 0$

when $x^3 = 8$

$\therefore x = 2$

$f(2) = 4\left(\frac{1}{4}\right) + 2 = 3$

$f(1) = 4 + 1 = 5$

$f(4) = \frac{4}{16} + 4 = \frac{17}{4}$

Max = 5, min = 3

(23) $\log_2(3x+7) = 3 + \log_2(x-1)$

$\log_2 \frac{3x+7}{x-1} = 3$

$\frac{3x+7}{x-1} = 2^3$

$3x+7 = 8(x-1)$

$3x+7 = 8x-8$

$7+8 = 8x-3x$

$15 = 5x$

$x = 3$

(24) $kx^2+3x+9k=0$

$a=k,\ b=3,\ c=9k$

$b^2-4ac \geq 0$ for real roots

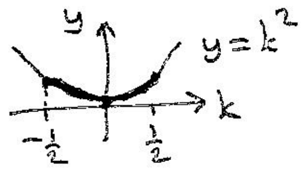
$3^2-4k(9k) \geq 0$

$9-36k^2 \geq 0$

$9 \geq 36k^2$

$k^2 \leq \frac{1}{4}$

Critical values $\pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$




$-\frac{1}{2} \leq k \leq \frac{1}{2}$

(25) (a) $D^2 = (2t-5)^2 + (t-10)^2$

$= 4t^2-20t+25+t^2-20t+100$

$= 5t^2-40t+125$

$D = \sqrt{5t^2-40t+125}$

(b) $D = (5t^2-40t+125)^{1/2}$

$\frac{dD}{dt} = \frac{1}{2}(5t^2-40t+125)^{-\frac{1}{2}}(10t-40)$

$D'(5) = \frac{1}{2}(125-200+125)^{-\frac{1}{2}}(50-40)$

$= \frac{1}{2}(50^{-\frac{1}{2}})(10)$

$= \frac{5}{\sqrt{50}} > 0$

$\therefore$ increasing.

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2015 Higher Maths

Old Specification

Paper 2

(1)(a) $m_{AB} = \frac{-5-7}{-1+5} = \frac{-12}{4} = -3$

$\therefore m_{alt} = \frac{1}{3}$

$y-3 = \frac{1}{3}(x-13)$

$3(y-3) = x-13$

$3y-9 = x-13$

$3y = x-4$

$4 = x-3y$

$x-3y=4$

(b) Midpoint M of AC

$= \left(\frac{-5+13}{2}, \frac{7+3}{2}\right)$

$= (4,5)$

$m_{BM} = \frac{5--5}{4--1} = \frac{10}{5} = 2$

$y=2x+c$

$5=2(4)+c$

$c=-3$

$y=2x-3$

(c) $\begin{cases} x-3y=4 & (1) \\ y=2x-3 & (2) \end{cases}$

Subs (2) into (1):

$x-3(2x-3)=4$

$x-6x+9=4$

$-5x=-5$

$x=1$

$\therefore y=2(1)-3=-1$

Point of intersection $(1,-1)$

(2) (a) $f(g(x)) = f((1+x)(3-x)+2)$

$= 10+(1+x)(3-x)+2$

$= (1+x)(3-x)+12$

(b) $f(g(x)) = 3+2x-x^2+12$

$= -x^2+2x+15$

$= -1(x^2-2x)+15$

$= -(x^2-2x+1-1)+15$

$= -(x^2-2x+1)+1+15$

$= -(x-1)^2+16$

② (c) $h(x) = \dfrac{1}{-(x-1)^2+16}$

Disallowed values are solutions of

$-(x-1)^2 + 16 = 0$

$(x-1)^2 = 16$

$x - 1 = \pm 4$

$x = -4+1,\ x = 4+1$

$x = -3,\ x = 5$

③ (a) $t_2 = \frac{3}{4}(13) + 13$

$= \frac{7}{4}(13)$

$= \frac{91}{4}$ or $22.75$

(b) Frog: $L = \dfrac{32}{1-\frac{1}{3}}$

$= 48$

The frog will not escape, as $48 < 50$.

Toad: $L = \dfrac{13}{1-\frac{3}{4}}$

$= 52$

The toad will escape, as $52 > 50$.

④ (a) $f(x) = g(x)$

$\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$

$-\frac{1}{2}x + 3 = -\frac{3}{2}x + 5$

$-3x + 18 = -9x + 30$

$6x = 12$

$x = 2$

(b) $f(x) - h(x)$

$= \left(\frac{1}{4}x^2 - \frac{1}{2}x + 3\right) - \left(\frac{3}{8}x^2 - \frac{9}{4}x + 3\right)$

$= -\frac{1}{8}x^2 + \frac{7}{4}x$

Area $= 2\int_0^2 \left(-\frac{1}{8}x^2 + \frac{7}{4}x\right) dx$

$= 2\left[-\frac{1}{24}x^3 + \frac{7}{8}x^2\right]_0^2$

④ (c) continued.

$= 2\left\{\left(-\frac{8}{24} + \frac{28}{8}\right) - (0)\right\}$

$= 2\left(-\frac{1}{3} + \frac{7}{2}\right)$

$= 2\left(-\frac{2}{6} + \frac{21}{6}\right)$

$= 2\left(\frac{19}{6}\right)$

$= \frac{19}{3}$ units$^2$

⑤ (a) Centre $C_1(-3,-5)$

$C_1C_2 = \sqrt{12^2 + 16^2}$

$= 20$

Radius of $C_1$

$= \sqrt{3^2 + 5^2 - 9}$

$= 5$

$\therefore$ radius of $C_2 = 20 - 5$

$= 15$

(b) Centre of $C_3$ divides $C_1C_2$ in the ratio $15:5 = 3:1$

$\therefore$ centre of $C_3$ is

$\left(-3 + \frac{3}{4}(9 - -3),\ -5 + \frac{3}{4}(11 - -5)\right)$

$= \left(-3 + \frac{3}{4}(12),\ -5 + \frac{3}{4}(16)\right)$

$= (-3 + 9,\ -5 + 12)$

$= (6, 7)$

$(x-6)^2 + (y-7)^2 = 20^2$

$(x-6)^2 + (y-7)^2 = 400$

⑥ (a) $\underset{\sim}{p} \cdot (\underset{\sim}{q} + \underset{\sim}{r}) = \underset{\sim}{p} \cdot \underset{\sim}{q} + \underset{\sim}{p} \cdot \underset{\sim}{r}$

$= |\underset{\sim}{p}||\underset{\sim}{q}|\cos 60^\circ + |\underset{\sim}{p}||\underset{\sim}{r}|\cos 90^\circ$

$= 3 \times 3 \times \frac{1}{2} + 0$

$= \frac{9}{2}$

(b) $\overrightarrow{EC} = -\underset{\sim}{q} + \underset{\sim}{p} + \underset{\sim}{r}$

⑥ (c) $\vec{AE} \cdot \vec{EC} = 9\sqrt{3} - \frac{9}{2}$

$$\underline{q} \cdot (-\underline{q} + \underline{p} + \underline{r}) = 9\sqrt{3} - \frac{9}{2}$$

$$-\underline{q}\cdot\underline{q} + \underline{q}\cdot\underline{p} + \underline{q}\cdot\underline{r} = 9\sqrt{3} - \frac{9}{2}$$

$$-3\times3\cos0^\circ + 3\times3\cos60^\circ + 3|\underline{r}|\cos30^\circ = 9\sqrt{3} - \frac{9}{2}$$

$$-9 + \frac{9}{2} + \frac{3\sqrt{3}}{2}|\underline{r}| = 9\sqrt{3} - \frac{9}{2}$$

$$\frac{3\sqrt{3}}{2}|\underline{r}| = 9\sqrt{3} - \frac{9}{2} + 9 - \frac{9}{2}$$

$$= 9\sqrt{3}$$

$$\frac{3}{2}|\underline{r}| = 9$$

$$|\underline{r}| = \frac{2}{3}(9)$$

$$= 6$$

⑦ (a) $\int (3\cos2x + 1)\,dx$

$$= \frac{3}{2}\sin2x + x + c$$

(b) $LHS = 3\cos2x + 1$

$$= 3(\cos^2x - \sin^2x) + (\sin^2x + \cos^2x)$$

$$= 3\cos^2x - 3\sin^2x + \sin^2x + \cos^2x$$

$$= 4\cos^2x - 2\sin^2x$$

$$= RHS$$

(c) $\int (\sin^2x - 2\cos^2x)\,dx$

$$= -\frac{1}{2}\int (-2\sin^2x + 4\cos^2x)\,dx$$

$$= -\frac{1}{2}\int (3\cos2x + 1)\,dx$$

$$= -\frac{1}{2}\left(\frac{3}{2}\sin2x + x\right) + c$$

$$= -\frac{3}{4}\sin2x - \frac{1}{2}x + c$$

⑧ $k\sin(1.5t - a)$

$$= k\sin1.5t\cos a - k\cos1.5t\sin a$$

$k\cos a = 36$

$k\sin a = 15$

$$k = \sqrt{36^2 + 15^2}$$

$$= \sqrt{1521}$$

$$= 39$$

$$\tan a = \frac{15}{36}$$

$$a = \tan^{-1}\frac{15}{36}$$

$$\approx 0.395$$

Ans: $39\sin(1.5t - 0.395)$

$h = 100$

$$36\sin1.5t - 15\cos1.5t + 65 = 100$$

$$39\sin(1.5t - 0.395) + 65 = 100$$

$$39\sin(1.5t - 0.395) = 35$$

$$\sin(1.5t - 0.395) = \frac{35}{39}$$

$$1.5t - 0.395 = 1.114,\ \pi - 1.114$$

$$1.5t - 0.395 = 1.114 \text{ or } 2.028$$

$$1.5t = 1.114 + 0.395 \text{ or } 2.028 + 0.395$$

$$1.5t = 1.509 \text{ or } 2.423$$

$$t = \frac{1.509}{1.5} \text{ or } \frac{2.423}{1.5}$$

$$t = 1.006 \text{ or } 1.615$$