Higher 2014 Paper 1

1. $U_3 = \frac{1}{3}(15) + 1 = 6$ **(C)**

$U_4 = \frac{1}{3}(6) + 1 = 3.$

2. $m_{CT} = \frac{2+1}{1-3} = \frac{3}{-2}$

if $\perp$, $\frac{3}{-2} \times \left(\frac{2}{3}\right) = 1$ **(B)**

3. $\log_4 12 - \log_4 x = \log_4 6$

$\log_4 12 - \log_4 6 = \log_4 x$ **(A)**

$\log_4 2 = \log_4 x$

4. $R\cos x \cos \alpha + R\sin x \sin \alpha$ **(D)**

$= 3\sin x - 4\cos x$

5. $\int (2x+9)^5 \, dx$

$= \frac{1}{12}(2x+9)^6 + C$ **(D)**

($\div$ new power and 2)

6. $\begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix} = \begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$ **(A)**

7. $\sin 2A = 2\sin A \cos A$

$= 2\left(\frac{3}{\sqrt{34}}\right)\left(\frac{5}{\sqrt{34}}\right)$ **(C)**

$= \frac{30}{34} = \frac{15}{17}$

8. $\frac{1}{2}(4-9x^4)^{-1/2} \cdot -36x^3$ **(D)**

$= -18x^3(4-9x^4)^{-1/2}$

9. Values - all x 5   **(B)**

10. $-1 < k-2 < 1$ **(C)**
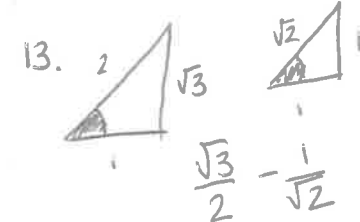
$1 < k < 3$

11. on $f(x)$ $(2,3)$ $(5,0)$ **(C)**

double y $(2,6)$ $(5,0)$

up 1 $(2,7)$ $(5,1)$

12. $x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$ **(A)**

$x = -8$ $x = 2$

13.

$\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$ **(A)**

14. $-6 + 2k + 5k =$ **(D)**

$7k = 6$

$k = \frac{6}{7}$

15. $k(x+1)(x-2) = y$ **(C)**

$k(1)(-2) = -8$

$k = 4.$

16. $\underline{a}\cdot\underline{a} + 2\underline{a}\cdot\underline{b}$ **(C)**

$1 + \frac{4}{3} = \frac{7}{3}$

17. $3x^2 + 6px + 3p^2 + q$

$= 3x^2 + 12x + 17.$ **(B)**

$p = 2, \quad q = 5$

18. $1 - 2\sin^2 x = \cos 2x$.

$\cos(2\times15) = \cos 30$ **(C)**

$= \frac{\sqrt{3}}{2}$

19. $\overrightarrow{ST} + \overrightarrow{TW} = -\underline{v} - \underline{u} - \underline{v}$ **(A)**

20. $2 - \log_5 \frac{1}{25}$ **(D)**

$2 - \frac{\log_5 5}{}(-2)$

21. $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2 = 0$ at SPs

so $3x(2-x) = 0$

$\underline{x=0}$ or $\underline{x=2}$

$y = 0$ $y = 12-8$

                $y = 4.$

	$\to$ 0 $\to$ 2 $\to$
$3x$	$-$ 0 $+$ 0 $-$
$(2-x)$	

min at (0,0)   max at (2,4)
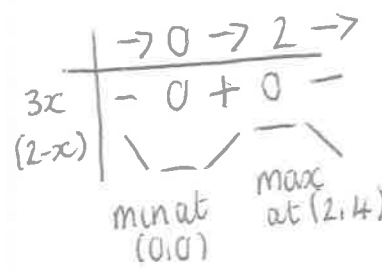
on x-axis, $y=0$

$x^2(3-x) = 0$

$x = 3$

on y axis, $x = 0$, $y = 0$

22.

-1	6	7	a	b
		-6	-1	$1-a$
	6	1	$a-1$	$1-a+b$

$= 0$ factor

2	6	7	a	b
		12	38	$2a + 76$
	6	19	$a + 38$	$2a + b + 76$

$= 72$

$2a + b + 76 = 72$

$2a + b = -4$

$+$

$a - b = 1$

$3a = -3$

$a = -1$

$a - b = 1 \quad \rightarrow \quad -b = 2$

$-1 - b = 1 \quad \rightarrow \quad b = -2$

b) $(x+1)(6x^2+x-2)$

$(x+1)(3x+2)(2x-1)$

$$\begin{array}{c|c|c}6x & 3x & 1\ 2 \\ \hline x & 2x & 2\ 1\end{array}$$

23a) $y=3x-5$ $x^2+y^2+2x-4y-15=0$

$x^2+(3x-5)^2+2x-4(3x-5)-15=0$

$\therefore x^2+9x^2-30x+25+2x-12x+20-15=0$

$\therefore 10x^2-40x+30=0$
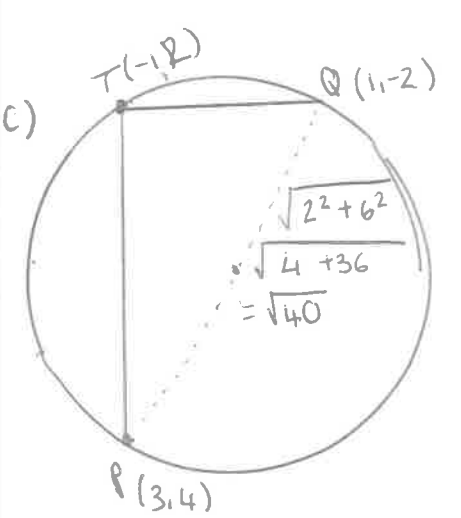
$\therefore 10(x^2-4x+3)=0$

$(x-3)(x-1)=0$

$x=3$ $x=1$

$y=3(3)-5$
$=9-5$
$=4$

$y=3(1)-5$
$=-2$

$P(3,4)$ $Q(1,-2)$

b) $T(-1,2)$ $m_{PT}=\frac{4-2}{3+1}=\frac{2}{4}=\frac{1}{2}$

$m_{QT}=\frac{-2-2}{1+1}=\frac{-4}{2}=-2$

$\frac{1}{2}x-2=-1$

$\therefore PT \perp QT$

c)

$r=\frac{1}{2}\sqrt{40}$
$=\frac{1}{2}(2\sqrt{10})$
$=\sqrt{10}$

$\sqrt{2^2+6^2}$
$\sqrt{4+36}$
$=\sqrt{40}$

$M_{PQ}=(\frac{4}{2},\frac{2}{2})$
$=(2,1)$

$(x-2)^2+(x-1)^2=10$

24. $y=ka^x$ A(6,5)   B(0,2)

$m_{AB}=\frac{5-2}{6-0}=\frac{1}{2}$

$\log_a y=\frac{1}{2}x+2$

$y=9^{\frac{1}{2}x+2}$

$y=9^{\frac{1}{2}x}\cdot 9^2$

$a=3$ $R=81$

Higher 2014 Paper 2
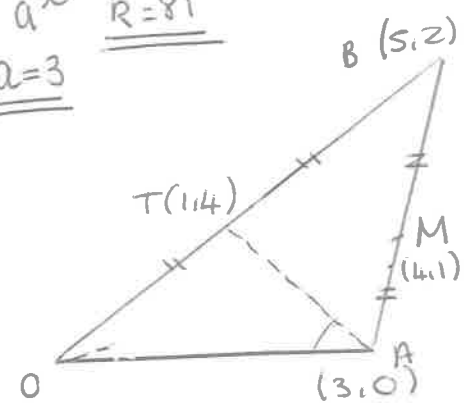
1. $M_{AB}=(\frac{3+5}{2},\frac{0+2}{2})=(4,1)$

$m_{AB}=\frac{y_A-y_B}{x_A-x_B}=\frac{2-0}{5-3}=1$

if $\perp$, $m_1 m_2=-1$

$\therefore m_{cm}=-1$

$y-1=-1(x-4)$
$y=-x+5$

b) $y=-2x+6$

$-x+5=-2x+6$

$x=1$

$y=-2(1)+6$
$y=-2+6$
$y=4$

$T(1,4)$

c) $\tan\theta=2$

$\theta=63.434...$

$\theta=63°$

2. $y = x^4 - 2x^3 + 5$ at $x=2$, $y = 2^4 - 2(2^3) + 5$

$\dfrac{dy}{dx} = 4x^3 - 6x^2$ $= 16 - 16 + 5$

$= 5$

at $x = 2$ $m = 8$, $(a,b) = (2,5)$

$\dfrac{dy}{dx} = 4(2^3) - 6(2^2)$ $y - 5 = 8(x-2)$

$= 4(8) - 6(4)$ $y - 5 = 8x - 16$

$y = 8x - 11$

$= 32 - 24$

$= 8$

3. $f(x) = x(x-1) + q$ $g(x) = x+3.$ $a = 1$
$b = 5$
$c = 6 + q$

$f(x+3) = (x+3)[x+3-1] + q$

$= (x+3)(x+2) + q$ $b^2 - 4ac = 0$

$= x^2 + 5x + 6 + q$ $5^2 - 4(1)(6+q) = 0$

$25 - 24 - 4q = 0$

$1 - 4q = 0$

$-4q = -1$

$q = \dfrac{1}{4}$

4a) $C(11,12,6)$ $D(8,8,4)$

b) $\vec{CB} = \underline{b} - \underline{c}$ $\vec{CD} = \underline{d} - \underline{c}$

$= \begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$ $|\vec{CB}| = \sqrt{64+16}$

$= \begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix}$ $= \sqrt{80}$

$|\vec{CD}| = \sqrt{9+16+4}$

$= \sqrt{29}$

c) $\cos BCD = \dfrac{\vec{CB} \cdot \vec{CD}}{|\vec{CB}||\vec{CD}|} = \dfrac{0+32+8}{\sqrt{80}\sqrt{29}}$

$\angle BCD = \cos^{-1}\left(\dfrac{40}{\sqrt{80}\sqrt{29}}\right) = 33.854\ldots = \underline{\underline{33.9^\circ}}$

5. $\displaystyle\int_{4}^{t} (3x+4)^{-\frac{1}{2}} \, dx = 2$

$= \left[\dfrac{2}{3}(3x+4)^{1/2} \right]_{4}^{t}$

$\Rightarrow \dfrac{2}{3}\sqrt{3t+4} - \dfrac{2}{3}\sqrt{3(4)+4} = 2$

$\dfrac{2}{3}\sqrt{3t+4} - \dfrac{8}{3} = 2$

$2\sqrt{3t+4} - 8 = 6$

$2\sqrt{3t+4} = 14$

$\sqrt{3t+4} = 7$

$3t+4 = 49$

$3t = 45$

$\underline{\underline{t = 15}}$

$\dfrac{4z \; 2x \;|\; 31}{x \; 2x \;|\; 13}$

6. $\sin x - 2\cos 2x = 1$

$\therefore \sin x - 2(1 - 2\sin^2 x) = 1$

$\sin x - 2 + 4\sin^2 x = 1$

$4\sin^2 x + \sin x - 3 = 0$

$(4\sin x - 3)(\sin x + 1) = 0$
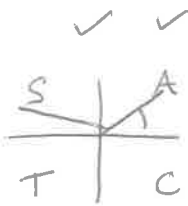
$\sin x = \dfrac{3}{4}$ $\sin x = -1$

(PTO)→

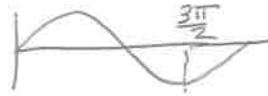**Q6 ctd:**

$\sin x = \dfrac{3}{4}$

$x = \sin^{-1}(3/4)$

$= 0.848\ldots$

$\underline{\underline{x = 0.8, \ 2.3}}$

 (S A / T C quadrant diagram with ✓ ✓)

$\sin x = -1$

$x = \dfrac{3\pi}{2}$

 (sine curve with $\frac{3\pi}{2}$ marked)

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7. $\underline{\text{limits}}$

$6x - x^2 = 2x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad x = 4$

$\displaystyle\int_{0}^{4} 4x - x^2 \, dx$

$= \left[2x^2 - \dfrac{x^3}{3} \right]_{0}^{4}$

$= \left[2(16) - \dfrac{64}{3} \right] - 0$

$= 32 - \dfrac{64}{3}$

$= \dfrac{96}{3} - \dfrac{64}{3}$

$= \dfrac{32}{3} \text{ units}^2 \times 300 = 3200 \, m^2.$

$6x - x^2 - 2x$

$= 4x - x^2$

b) $y = 6x - x^2$

$\dfrac{dy}{dx} = 6 - 2x = 2$

$-2x = -4$

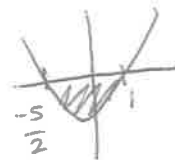$\underline{x = 2}$

$y = 6(2) - 2^2$

$= 12 - 4$

$= 8.$

eqn tgt

$y - 8 = 2(x - 2)$

$y - 8 = 2x - 4$

$\underline{\underline{y = 2x + 4}}$

$\displaystyle\int_{0}^{2} 2x + 4 - (6x - x^2) \, dx$

$= \int 2x + 4 - 6x + x^2$

$= \int_{0}^{2} x^2 - 4x + 4 \, dx$

$= \left[\dfrac{x^3}{3} - 2x^2 + 4x \right]_{0}^{2}$

$= \left[\dfrac{8}{3} - 8 + 8 \right] - [0]$

$= \underline{\dfrac{8}{3} \text{ units}.}$

8. $x^2 + y^2 - 2px - 4py + 3p + 2.$

centre $(p, 2p)$ ∴ $r = \sqrt{p^2 + 4p^2 - 3p - 2}$

 (parabola sketch with $-\frac{5}{2}$ and 1 marked)

$\dfrac{-5}{2} < p < 1$

∴ $5p^2 - 3p - 2 > 0$

$(5p + 2)(p - 1) = 0$

$p = \dfrac{-2}{5} \quad p = 1$

9. $V(t) = 8\cos\left(2t - \frac{\pi}{2}\right)$

$a(t) = V'(t)$

$\quad = 16\sin\left(2t - \frac{\pi}{2}\right)$

$a(10) = -16\sin\left(20 - \frac{\pi}{2}\right)$

$\quad = 6.529\ldots$

$\quad = \underline{6.53}$

$a(10) > 0 \therefore$ increasing

c) $S(t) = \displaystyle\int V(t)$

$\quad = \displaystyle\int 8\cos\left(2t - \frac{\pi}{2}\right) dt$

$\quad = \left[\dfrac{8}{2}\sin\left(2t - \frac{\pi}{2}\right) + C\right]$

at t=0: $\quad 4\sin\left(\frac{-\pi}{2}\right) + C = 4$

$\quad\quad\quad\quad C = 4 - 4\sin\left(\frac{-\pi}{2}\right)$

$\quad\quad\quad\quad C = 4 - 4(-1)$

$\quad\quad\quad\quad C = 8$

$\therefore\ S(t) = \underline{\underline{4\sin\left(2t - \frac{\pi}{2}\right) + 8}}$