Higher 2013 Paper 1

1. $3(x^2+1)-4$ (A)

$3x^2+3-4$

2. $2x-4$ at $x=5$ (B)

$=10-4$

$=6$

3. $4^2-4(2)(5)$ (B)

$16-40$

$=-24$

4. Stretch 8 (A)

Period π

down 1

5. $y+1 = \frac{-5}{3}(x+2)$ (D)

$3y+3=-5x-10$

$5x+3y+13=0$

6. (C)

2	1	3	-5	-6
	0	2	10	10
	1	5	5	4

7. $\int 3x^2+2x$ (B)

8. $-1<0.1<1$ (C)

$\therefore$ limit exists $\therefore$ (2) true

$11=0.1u_0+8$

$3=0.1u_0$

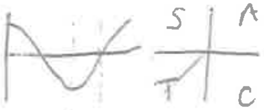
$u_0=30$

9. $\sin 2x = 2\sin x\cos x$ (A)

$=2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$

$=\frac{4}{5}$

10. (D)



11. reflect over x-axis (B)

slide right

12. $\underline{f}+\underline{g} = 5\underline{i}+4\underline{j}+5\underline{k}$ (C)

$|\underline{f}+\underline{g}| = \sqrt{66}$

13. $x^2-7x+12=0$ (A)

$(x-3)(x-4)=0$

$x=3,\ x=4$

14. $a.a + a.b$ (B)

$|a||a|\cos 0 + 5$

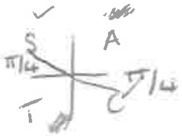
$=(3)(3)(1)+5$

$=14$

15.

$\tan\frac{x}{2}=-1$

$\tan^{-1}(1)=\frac{\pi}{4}$



$\frac{x}{2}=\frac{3\pi}{4},\ \frac{7\pi}{4}$

$x=\frac{3\pi}{2},\ \cancel{\frac{7\pi}{2}}$ $\left(\frac{7\pi}{2}>2\pi\right)$

(C)

16. $\int(1-6x)^{-1/2}dx$ (C)

$=\frac{(1-6x)^{1/2}}{\left(\frac{1}{2}\right)(-6)}+C$

$=\frac{(1-6x)^{1/2}}{-\frac{6}{2}}+C$

$=\frac{-1(1-6x)^{1/2}}{3}+C$

17. $y=kx(x+a)^2$ (C)

roots at $(-2,0)$ $(0,0)$

$\therefore a=2.$

at $(1,3)$

$3=k(1)(1+2)^2$

$3=k(9)$

$\therefore k=\frac{1}{3}$

18. $y=\sin(x^2-3)$ (D)

$\frac{dy}{dx}=\cos(x^2-3)\times 2x$

$=2x\cos(x^2-3)$

19. $1-2x-3x^2>0$ (B)

$(1-3x)(1+x)>0$

$\therefore 1-3x=0$ $\quad 1+x=0$

$1=3x$ $\quad x=-1$

$\therefore x=\frac{1}{3}$

$-1<x<\frac{1}{3}$

20. $\log_3 y = 2x+0$ (D)

(as intercept @ $(0,0)$)

$\therefore y=3^{2x}$

$y=(9)^x$

$y=9^x$

21. $2x^2+12x+1$

$a(x+b)^2+c$

$=a(x^2+2bx+b^2)+c$

$=ax^2+2abx+b^2+c$

equate coefficients

$\therefore ax^2=2x^2$

$a=2$

$12x=2abx$

$6=ab$

$6=(2)(b)$

$b=3$

$b^2+c=1$

$3^2+c=1$

$9+c=1$

$c=-8$

$\therefore 2x^2+12x+1=2(x+3)^2-8.$

22. $x^2 + y^2 + 2x + 4y - 27 = 0$

Centre, $(-1, -2)$

radius$_1$ $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-1)^2 + (-2)^2 + 27}$

$= \sqrt{1 + 4 + 27}$

$= \sqrt{32}$

b) $C(-1,-2)$ $P(3,2)$

$m_{CP} = \dfrac{y_c - y_p}{x_c - x_p}$

$= \dfrac{-2-2}{-1-3}$

$= \dfrac{-4}{-4} = 1$

$\therefore m_{tgt} = -1$ as $m_1 m_2 = -1$ since $\perp$

$m = -1$
$(a,b) = (3,2)$

$y - b = m(x - a)$

$y - 2 = -1(x - 3)$

$y - 2 = -1x + 3$

$y = -x + 5$

c) $C_2(10,-1)$

$r_2 = \dfrac{\sqrt{32}}{2}$

$= \dfrac{\sqrt{16}\sqrt{2}}{2}$

$= \dfrac{4\sqrt{2}}{2}$

$= 2\sqrt{2}$

$(x-10)^2 + (y+1)^2 = 8$

$x^2 - 20x + 100 + y^2 + 2y + 1 = 8$

$x^2 + y^2 - 20x + 2y + 101 = 8$

$x^2 + y^2 - 20x + 2y + 93 = 0$

d) If tangent, 1 pt of contact - 1 solution / root

$y = -x + 5$
or
$y = 5 - x$

$x^2 + (5-x)^2 - 20x + 2(5-x) + 93 = 0$

$x^2 + 25 - 10x + x^2 - 20x + 10 - 2x + 93 = 0$

$\therefore 2x^2 - 32x + 128 = 0$

$\therefore x^2 - 16x + 64 = 0$

$(x-8)(x-8) = 0$

$x = 8$

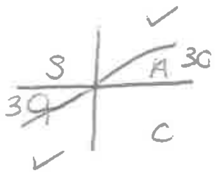
1 repeated root
$\therefore$ tangent.

23 $\sqrt{3}\sin x - \cos x = k\sin x\cos a - k\cos x\sin a$

$\therefore k\cos a = \sqrt{3}$
$k\sin a = 1$

$\tan a = \dfrac{k\sin a}{k\cos a}$

$= \dfrac{1}{\sqrt{3}}$



$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$

$x = 30°$

$\therefore k^2 = (\sqrt{3})^2 + (1)^2$

$k = \sqrt{3+1}$

$k = 2.$

$\therefore \sqrt{3}\sin x - \cos x = 2\sin(x - 30°)$

24.

$\vec{AT} = \underline{t} - \underline{a}$

$= \begin{pmatrix}3\\2\\5\end{pmatrix} - \begin{pmatrix}-7\\-8\\1\end{pmatrix}$

$= \begin{pmatrix}10\\10\\4\end{pmatrix} = 2\begin{pmatrix}5\\5\\2\end{pmatrix}$

$\vec{TB} = \underline{b} - \underline{t}$

$= \begin{pmatrix}18\\17\\11\end{pmatrix} - \begin{pmatrix}3\\2\\5\end{pmatrix}$

$= \begin{pmatrix}15\\15\\6\end{pmatrix} = 3\begin{pmatrix}5\\5\\2\end{pmatrix}$

$\vec{AT}$ // to $\vec{TB}$,
B common pt
$\therefore$ collinear

a) T splits AB in ratio 2:3.

b) $C(x,0,0)$

if TB perpendicular to TC then $\vec{TB}\cdot\vec{TC}=0$

$$\vec{TC}=\begin{pmatrix}x-3\\0-2\\0-5\end{pmatrix}\qquad \vec{TB}=\begin{pmatrix}15\\15\\6\end{pmatrix}$$

$$=\begin{pmatrix}x-3\\-2\\-5\end{pmatrix}$$

$$\vec{TC}\cdot\vec{TB}=15(x-3)+(-2)(15)+(-5)(6)$$

$$0=15x-45-30-30$$

$$0=15x-105$$

$$15x=105$$

$$\underline{x=7}\qquad\qquad \underline{\underline{C(7,0,0)}}$$

Higher 2013 Paper 1

1. $u_1 = 4$

$u_2 = 7$

$u_3 = 16$

$$7 = 4m + C$$
$$16 = 7m + C$$
$$-9 = -3m$$
$$\therefore m = 3$$

$$4m + C = 7$$
$$4(3) + C = 7$$
$$12 + C = 7$$
$$C = -5$$

2. $m = 1/2$

$(a,b) = (5,6)$

since RQ ⊥ QP. If ⊥, $m_1 m_2 = -1$

$$-2 \times \boxed{\tfrac{1}{2}} = -1$$

$$m_{QP} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{6-2}{5-7} = \frac{4}{-2} = -2$$

$$y - 6 = \tfrac{1}{2}(x-5)$$
$$2y - 12 = x - 5$$
$$2y = x + 7$$
$$y = \tfrac{1}{2}x + \tfrac{7}{2}$$

rearrange for (b)

$$-x + 2y = 7$$

b) $x + 3y = 13$

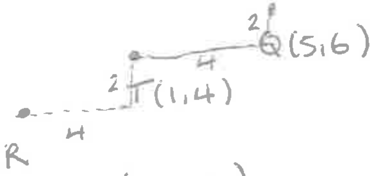
$+ \; -x + 2y = 7$

$$5y = 20$$
$$y = 4$$

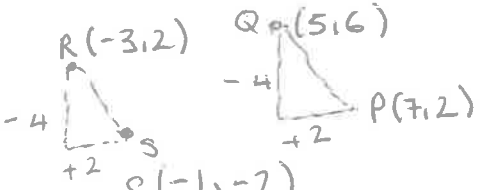
$$-x + 2y = 7$$
$$-x + 2(4) = 7$$
$$-x + 8 = 7$$
$$-x = -1$$
$$x = 1$$

T (1,4)

c)



R(−3,2)



S(−1,−2)

3.

$$\begin{array}{r|rrrr} 1 & 1 & 2 & 1 & -5 \\ & 0 & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & \boxed{0} \end{array}$$

∴ factor

$\therefore (x-1)(x^2+4x+5)$ ← no further solution

= [illegible]

b) $y = x^4 + 4x^3 + 2x^2 - 20x + 3$

$$\frac{dy}{dx} = 4x^3 + 12x^2 + 4x - 20 = 0 \text{ at max/min}$$

$$4(x^3 + 3x^2 + x - 5) = 0$$

$$4\,(x-1)(x^2+4x+5) = 0$$

$\downarrow$ $x = 1$

$\downarrow$ no solutions as $b^2 - 4ac < 0$.

$\frac{dy}{dx}$	→	1	→
$(x-1)$ (x^2+4x+5)	− + −ve	0	+ + +ve
	\	—	/

=

min at $x = 1$

4.

Pts of intersection

let $x^3+3x^2+2x+3 = 2x+3$

∴ $x^3+3x^2 = 0.$

$x^2(x+3) = 0$

$x = 0$ $\quad x = -3$

at $x = -3$

$y = 2(-3)+3$

$= -6+3$

$= -3$

$B(-3,-3).$

area (Curve - line)

$$\int_{-3}^{0} x^3+3x^2\,dx$$

$$= \left[\frac{x^4}{4}+x^3\right]_{-3}^{0}$$

$$= \left[\frac{0}{4}+0\right] - \left[\frac{(-3)^4}{4}+(-3)^3\right]$$

$$= -\left[\frac{81}{4}-27\right]$$

$$= -\left[\frac{81}{4}-\frac{108}{4}\right]$$

$$= -\left[\frac{-27}{4}\right]$$

$$= \frac{27}{4}\text{ units}^2$$

5. $\log_5(3-2x)+\log_5(2+x) = 1$

$\log_5(3-2x)(2+x) = 1$

∴ $(3-2x)(2+x) = 5^1$

∴ $6+3x-4x-2x^2 = 5$

∴ $2x^2+x-1 = 0$

∴ $(2x-1)(x+1) = 0$

$x = \frac{1}{2}$ $\quad x = -1$

6. $\int_{0}^{a} 5\sin 3x\,dx = \frac{10}{3}.$

$\left[-\frac{5}{3}\cos 3x\right]_{0}^{a} = \frac{10}{3}$

∴ $-\frac{5}{3}\cos 3a - \left(-\frac{5}{3}\cos 0\right) = \frac{10}{3}$

∴ $-\frac{5}{3}\cos 3a + \frac{5}{3}(1) = \frac{10}{3}.$

$-\frac{5}{3}\cos 3a = \frac{5}{3}.$

$-\cos 3a = 1.$

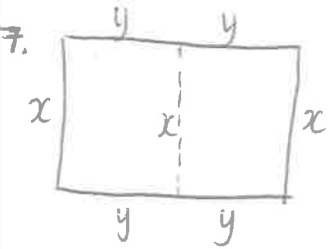
$\cos 3a = -1$

$3a = \cos^{-1}(-1)$

$3a = \pi$

$a = \frac{\pi}{3}$

7.



$L = 3x + 4y.$

Since $A = 2xy.$

$24 = 2xy.$

$\frac{24}{2x} = y$

$y = \frac{12}{x}$

$\therefore L = 3x + 4\left(\frac{12}{x}\right)$

$= 3x + \frac{48}{x} \Rightarrow 3x + 48x^{-1}$

b) $L'(x) = 3 - 48x^{-2} = 0$ at max/min

$\therefore 3 - \frac{48}{x^2} = 0$

$\frac{48}{x^2} = 3.$

$48 = 3x^2.$

$3x^2 - 48 = 0$

$3(x^2 - 16) = 0$

$3(x-4)(x+4) = 0$

$x = 4$ ~~$x = -4$~~

L'(x)	→	4	→
(x-4) (x+4)	-ve	0	+ve
	\	—	/

min at $x=4$

when $x = 4$,

$y = \frac{12}{4} = 3.$

$L = 3(4) + 4(3)$
$= 24m$

24 × 8.25.

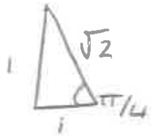
= 8.25 × 4 × 6
= 33.00 × 6
= £198.00.

$$\begin{array}{r} 8.25 \\ \times \quad 4 \\ \hline 33.00 \end{array}$$

8. $\sin 2x = 2\cos^2 x \qquad 0 < x < 2\pi$

$2\sin x\cos x - 2\cos^2 x = 0$

$2\cos x(\sin x - \cos x) = 0.$



$2\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

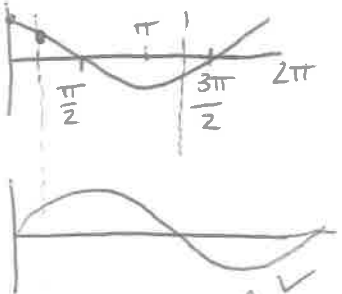
$\sin x - \cos x = 0$

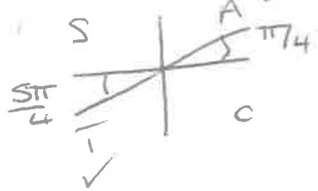
$\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$





9. $P_t = P_0 e^{-kt}$

$0.5 = e^{-25k}.$

$\log_e 0.5 = -25k$

$k = \frac{\log_e 0.5}{-25}$

$k = 0.028$ (2 sig figs)

b) Start at 100% : $P_0 = 100$, $t = 80$ days

$P = 100e^{-kt}$

$P = 100e^{-(0.028 \times 80)}$

$P = 100e^{-2.24}$

$P = 10.6$ remains

∴ 89.4% decrease.