# Higher 2012 Paper 1

1.
$u_1 = 3(1) + 4 = 7$

$u_2 = 3(7) + 4 = \underline{\underline{25}}$ (C)

2. $y = x^3 - 6x + 1$

$\frac{dy}{dx} = 3x^2 - 6$

at $x = -2$

$\frac{dy}{dx} = 3(-2)^2 - 6$

$= 12 - 6$

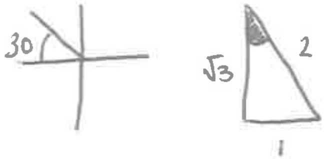
$\underline{\underline{= 6}}$ (D)

3. $x^2 - 6x + 14$

$\downarrow \div 2$ $\quad 3^2 = 9$

$\underline{\underline{(x-3)^2 + 5}}$ (B)

4. $\tan 150 = -\tan 30 = \frac{-1}{\sqrt{3}}$




(B)

5. $\cos 2A = \cos^2 A - \sin^2 A$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$ (A)

6. $u = 3x^{-2} + 2x^{3/2}$ (C)

$\frac{dy}{dx} = -6x^{-3} + 3x^{1/2}$

7. if ⊥, $\underline{u} \cdot \underline{v} = 0$

$\therefore (-3 \times 1) + (t) + (-2t) = 0$ (A)

$-3 + -t = 0$

$\underline{\underline{t = -3}}$

8. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

at $r = 2$.

$\frac{dV}{dr} = 4\pi(2^2)$ (C)

$= \underline{16\pi}$

9. → right by $\frac{\pi}{6}$ $\quad (x - \frac{\pi}{6})$

down by 1 $\quad y = \cos(x - \frac{\pi}{6}) - 1$ (B)

10. $\vec{RQ} + \vec{QT} + \vec{TP}$ (B)

$= -\underline{f} - \underline{g} + \underline{h}$

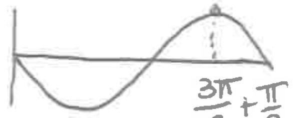
11. $\int \frac{1}{6} x^{-2}$

$= \frac{-1}{6} x^{-1} + C$ (D)

12. $2 - 3\sin(x - \frac{\pi}{3})$ right by $+\pi/3$.

max/min $\pm 3$.

add 2 → max = 5



negative singraph

$\frac{3\pi}{2} + \frac{\pi}{3}$

$= \frac{9\pi}{6} + \frac{2\pi}{6} = \frac{11\pi}{6}$ (B)

13. $y = k(x+2)(x+1)$ at $(0, 6)$

$6 = k(2)(1)$

$k = 3$ (D)

$\therefore y = 3(x+2)(x+1)$

14. $\int (2x-1)^{1/2} dx$

$= \frac{(2x-1)^{3/2}}{3/2 \times 2} + c$

$= \frac{(2x-1)^{3/2}}{3} + c$ (A)

15. $|\underline{u}| = 1$ since unit vector

$\sqrt{(3k)^2 + (-k)^2 + 0^2} = 1$

$9k^2 + k^2 = 1$

$10k^2 = 1$

$k = \frac{1}{\sqrt{10}}$ (D)

# 2012 Paper 1

16. $y = 3\cos^4 x = 3(\cos x)^4$

$\frac{dy}{dx} = -12\cos^3 x \cdot \sin x$

(C)

17. $\underline{a}\cdot(\underline{a}+\underline{b}) = 7$

$\underline{a}\cdot\underline{a} + \underline{a}\cdot\underline{b} = 7$

$(9+16+0) + \underline{a}\cdot\underline{b} = 7$

$25 + \underline{a}\cdot\underline{b} = 7$

$\underline{a}\cdot\underline{b} = 7 - 25$

$= -18$

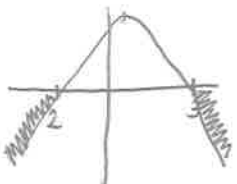
(D)

18.

- In between S and T, the graph is below $x$-axis $\Rightarrow f(x) < 0$.
- Before $q$, the graph has a negative gradient so $f'(x) < 0$.

(D)

19. $(3+x)(2-x) < 0$




$x < -2$

$x > 3$

(D)

20. $\dfrac{\log_b 9a^2}{\log_b 3a}$

$= \dfrac{\log_b (3a)^2}{\log_b (3a)}$

$= \dfrac{2\log_b (3a)}{\log_b (3a)}$

$= 2$

(A)

## Section B

a) $x - 4 = 0$

$x = 4$

$$\begin{array}{c|cccc} 4 & 1 & -5 & 2 & 8 \\ & 0 & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

$0 \therefore$ factor

b) $(x-4)(x^2 - x - 2)$

$(x-4)(x-2)(x+1)$

ii) $(x-4)(x-2)(x+1) = 0$

$x = 4, \quad x = 2, \quad x = -1$

b) $\displaystyle\int_0^{Q=2} x^3 - 5x^2 + 2x + 8 \, dx$

$= \left[\frac{x^4}{4} - \frac{5}{3}x^3 + x^2 + 8x\right]_0^2$

$= \left[\frac{2^4}{4} - \frac{5}{3}(2^3) + 2^2 + 8(2)\right] - [0]$

$= \frac{16}{4} - \frac{40}{3} + 4 + 16$

$= \frac{16}{4} - \frac{40}{3} + 22$

$=$

$= 24 - \frac{40}{3}$

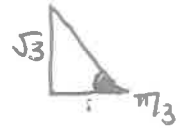
$= \frac{72}{3} - \frac{40}{3}$

$= \frac{32}{3}$ units$^2$

22a) $\cos x - \sqrt{3}\sin x = k\cos x\cos a - k\sin x\sin a$

$k\cos a = 1$

$k\sin a = \sqrt{3}$. $\quad \frac{s}{c} = t$




$\tan a = \frac{\sqrt{3}}{1}$

$a = \pi/3$.

$(k\sin a)^2 + (k\cos a)^2$

$k^2\sin^2 a + k^2\cos^2 a = (\sqrt{3})^2 + (1^2)$

$k^2(\sin^2 a + \cos^2 a) = 3 + 1$

$k^2 = 4 \quad \therefore \cos x - \sqrt{3}\sin x$

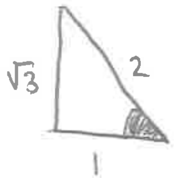
$k = 2 \quad = 2\cos(x + \pi/3)$.

b) on y axis, $x=0$

$y = 2\cos(\frac{\pi}{3})$

$= 2(\frac{1}{2})$

$\underline{\underline{= 1}}$




on $x$-axis, $y=0$

$2\cos(x + \frac{\pi}{3}) = 0$

$\cos(x + \frac{\pi}{3}) = 0$

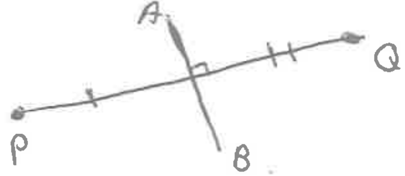
$x + \frac{\pi}{3} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x + \frac{2\pi}{6} = \frac{3\pi}{6}, \frac{9\pi}{6}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$.

$\boxed{(0,1) \quad (\frac{\pi}{6}, 0) \quad (\frac{7\pi}{6}, 0)}$

23a) $P(3,-3)$ $Q(-1,9)$




$m_{PQ} = \frac{y_P - y_Q}{x_P - x_Q}$

$= \frac{-3-9}{3-(-1)}$

$= \frac{-12}{4}$

$\underline{\underline{= -3}}$

If ⊥, $m_1 m_2 = -1$

$-3 \times \frac{1}{3} = -1$

$M_{PQ} = (1,3)$

$m_{AB} = \frac{1}{3}$

$l_1 =$

$y - 3 = \frac{1}{3}(x-1)$

$3y - 9 = x - 1$

$3y = x + 8$

$\underline{\underline{y = \frac{1}{3}x + \frac{8}{3}}}$

b) $m_{CD} = -3$

$(a,b) = (1,-2)$

$y+2 = -3(x-1)$

$y+2 = -3x+3$

$\underline{\underline{y = -3x+1}}$

c) let $\frac{1}{3}x + \frac{8}{3} = -3x + 1$

$x + 8 = -9x + 3$

$10x = -5$

$x = \frac{-5}{10}$

$\underline{\underline{x = \frac{-1}{2}}}$

$y = -3(\frac{-1}{2}) + 1$

$= \frac{3}{2} + 1$

$\underline{\underline{= \frac{5}{2}}}$

$\underline{\underline{(\frac{-1}{2}, \frac{5}{2})}}$

d) $d = \sqrt{(1+\frac{1}{2})^2 + (3 - 5/2)^2}$

$= \sqrt{(3/2)^2 + (1/2)^2}$

$= \sqrt{9/4 + 1/4}$

$=$

$= \sqrt{\frac{10}{4}}$

$\underline{\underline{= \frac{\sqrt{10}}{2} \text{ units}}}$

M $(1,3)$

$(\frac{-1}{2}, \frac{5}{2})$

# Higher 2012 Paper 2

1. $g(x) = x+4 \qquad f(x) = x^2+3$

$f(g(x)) = f(x+4) \qquad f(x+4) = (x+4)^2+3$

$= x^2+8x+16+3$

$= x^2+8x+19$

$f(x) = x^2+3 \qquad g(x) = x+4$

$g(f(x)) = f(x^2+3) \qquad g(x^2+3) = x^2+3+4$

$= x^2+7$

b) $x^2+8x+19 \pm x^2+7 = 0$

$2x^2+8x+26 = 0$

$2(x^2+4x+13) = 0$

$b^2-4ac$

$= 4^2-4(1)(13)$

$= 16-52$

$= -36$

$b^2-4ac < 0 \therefore$ no real roots.

2a) $y = 2x+5$

$x^2+y^2-6x-2y-30 = 0$

$x^2+(2x+5)^2-6x-2(2x+5)-30 = 0$

$x^2+4x^2+20x+25-6x-4x-10-30 = 0$

$5x^2+10x-15 = 0$

$5(x^2+2x-3) = 0$

$5(x+3)(x-1) = 0$

$x = -3 \qquad x = 1$

at $x=-3$, $y = 2(-3)+5 = -1$

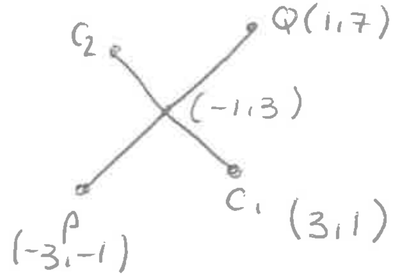
at $x = 1$ $y = 2(1)+5 = 7$

$P(-3,-1) \quad Q(1,7)$

$x^2+y^2-6x-2y-30 = 0$

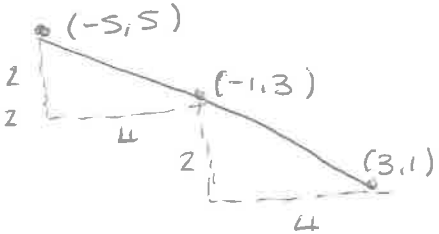
$2g = -6 \qquad 2f = -2$

$g = -3 \qquad f = -1$




Midpoint $PQ = (-1,3)$

$C_2 = (-5,5)$.




Circle 2 = $(x+5)^2+(y-5)^2 = r^2$

radius$_1$ = $\sqrt{(-3)^2+(-1)^2+30}$

$r^2 = 40$

$(x+5)^2+(x-5)^2 = 40$.

3. $f(x) = x^3-2x^2-4x+6$

$f'(x) = 3x^2-4x-4 = 0$ at max/min

| | | | |
|---|---|---|---|
| $3x$ | 1 | 2 | 4 |
| $x$ | 4 | 2 | 1 |

$-6x$, $+2x$

$(3x+2)(x-2) = 0$

$3x+2 = 0 \qquad x = 2$

$\cancel{x = -\frac{2}{3}}$ outwith domain (bounds)

$\cancel{f(-\tfrac{2}{3}) = (-\tfrac{2}{3})^3-2(-\tfrac{2}{3})^2-4(-\tfrac{2}{3})+6}$

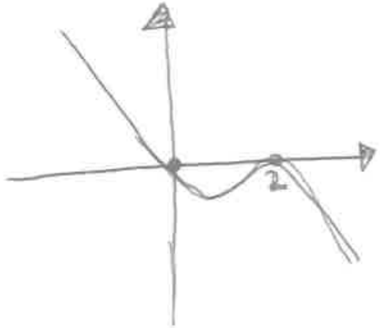
$f(2) = 2^3-2(2^2)-4(2)+6 = -2 \quad (2,-2)$

$f(3) = 3^3-2(3^2)-4(3)+6 = 3 \quad (3,3)$

$f(0) = 0^3-2(0^2)-4(0)+6 = 6 \quad (0,6)$

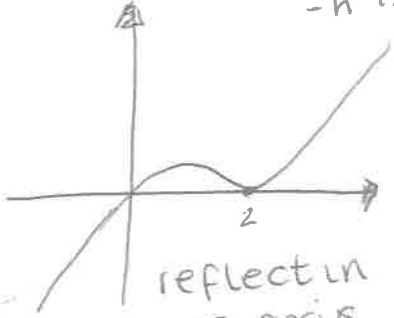
$\therefore$ max value $= 6$, min value $-2$.

4. *TPs become roots

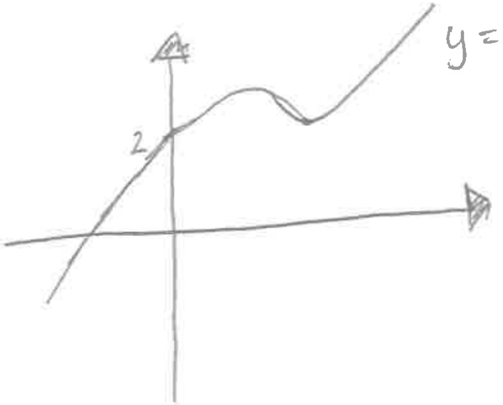



$y = h'(x)$

$-h'(x)$




reflect in $x$-axis
( $y$ becomes -ve )




$y = 2 - h'(x)$

( add 2 to function
slide up by 2 )

5. $\vec{BA} = \underline{a} - \underline{b}$

$= \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$|\vec{BA}| = \sqrt{2}$.

$\vec{BC} = \underline{c} - \underline{b}$

$= \begin{pmatrix} 4 \\ k \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ k+3 \\ -1 \end{pmatrix}$

$|\vec{BC}| = \sqrt{2^2 + (k+3)^2 + 1}$

$= \sqrt{4 + k^2 + 6k + 9 + 1}$

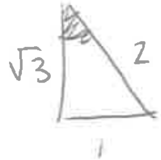
$= \sqrt{k^2 + 6k + 14}$

$\cos ABC = \dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$

$= \dfrac{(1)(2) + 0(k+3) + (-1)(-1)}{\sqrt{2} \times \sqrt{k^2+6k+14}}$

$= \dfrac{3}{\sqrt{2(k^2+6k+14)}}$

$\cos 30 = \dfrac{\sqrt{3}}{2}$




$\dfrac{3}{\sqrt{2k^2+12k+28}} = \dfrac{\sqrt{3}}{2}$

$\dfrac{9}{2k^2+12k+28} = \dfrac{3}{4}$

$36 = 3(2k^2 + 12k + 28)$

$12 = 2k^2 + 12k + 28$

$6 = k^2 + 6k + 14$

$k^2 + 6k + 8 = 0$

$(k+4)(k+2) = 0$

$\underline{k = -4}$ $\underline{k = -2}$

6. for $u_{n+1} = au_n + b$, if $-1 < a < 1$ then limit exists. Since $\sin x$ has max $= 1$ and min $-1$, limit exists.

b) $L = \frac{1}{2}\sin x$ $\therefore$ $\frac{1}{2}\sin x = \frac{1}{2}\sin x(\sin x) + \cos 2x$

$\frac{1}{2}\sin x = \frac{1}{2}\sin^2 x + \cos 2x$

$\frac{1}{2}\sin x = \frac{1}{2}\sin^2 x + 1 - 2\sin^2 x$

$\sin x = \sin^2 x + 2 - 4\sin^2 x$

$3\sin^2 x + \sin x - 2 = 0$

$(3\sin x - 2)(\sin x + 1) = 0$

$3\sin x - 2 = 0$

$\sin x = \frac{2}{3}$

$\sin^{-1}\left(\frac{2}{3}\right) = 0.73 \text{ (radians)}$

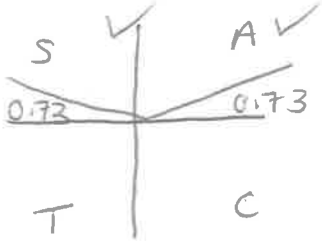
$x = 0.73, \pi - 0.73$

$= 0.73, 2.41.$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}.$




$$\boxed{\begin{array}{c} x = 0.73, 2.41, \frac{3\pi}{2} \\ \text{or} \\ x = 0.73, 2.41, 4.71 \end{array}}$$

Since $0 < x < \frac{\pi}{2}$, $x = 0.73.$

7a) $3^{(2-x)} = 4^x.$

$\log 3^{(2-x)} = \log 4^x.$

$(2-x)\log 3 = x\log 4$

$\frac{2-x}{x} = \frac{\log 4}{\log 3}$

$\frac{2}{x} - 1 = \frac{\log 4}{\log 3}$

$\frac{2}{x} = \frac{\log 4}{\log 3} + \frac{\log 3}{\log 3}$

$\frac{2}{x} = \frac{\log 12}{\log 3}$

$\frac{x}{2} = \frac{\log 3}{\log 12}$

$x = \frac{2\log 3}{\log 12}$

$x = \log 3^2$

$x = \frac{\log 9}{\log 12}$

Since $y = 4^x.$

when $x = \frac{\log_{10} 9}{\log_{10} 12}$

$= 0.88$

$y = 4^{0.88}$

$y = 3.39 \quad \therefore T(0.88, 3.39).$