2009 Paper 1

1. $u_2 = 3\times2+4 = 10$
$u_2 = 3\times10+4 = 34$ (A)

2. $M(1,5)$ $m_{ps} = \frac{5+2}{1+3} = \frac{7}{4}$ (D)

3. $\frac{dy}{dx} = 15x^2 - 12$
$m = 15(1) - 12 = \underline{3}$ (C)

4. $r = \sqrt{4^2+3^2+75}$
$= \underline{10}$ (B)

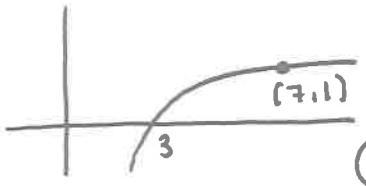
5. $ST = \sqrt{4^2+3^2} = 5$
$m = \frac{-1-3}{5-2} = \frac{-4}{3}$ (B)

6. $L = 0{\cdot}7L + 10$
$0{\cdot}3L = 10$
$L = \frac{10}{0{\cdot}3} = \frac{100}{3}$ (A)

7. $\cos 2x = 2\cos^2 x - 1$
$= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$
$= \frac{2}{5} - 1$
$= -3/5$ (A)

8. $f(x) = \frac{1}{4}x^{-3}$
$f'(x) = \frac{-3}{4}x^{-4}$ (D)

9. $x^2 + (2x)^2 = 5$
$5x^2 = 5$
$x = \pm 1$ (A)

10. 

(B)

11. $4\sin x - \sqrt{5} = 0$ $\sin x + 1 = 0$
$\sin x = \frac{\sqrt{5}}{4}$ $\sin x = -1$
↓ ↓
2 solutions 1 solution (B)

12. $b^2 - 4ac$
$(-1)^2 - 4(2)(-9)$
$= 1 + 72$
$= 73$ (C)

13. $k = \sqrt{1^2 + (\sqrt{3})^2}$
$= \underline{2}$
$\tan a = \frac{1}{\sqrt{3}}$
$a = 30$ (B)

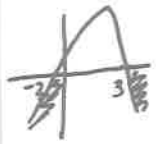
14. max/min 2 / -2
up 5 7 / 3
max 7, min 3. (C)

15. $m = \tan 60° = \frac{\sqrt{3}}{1}$ (A)

16. $\int_0^1 4x^3 - 9x^2\,dx$
$-\left[x^4 - 3x^3\right]_0^1$ (B)

17. $|A| = \sqrt{\left(\frac{-3}{5}\right)^2 + 0^2 + \left(\frac{4}{5}\right)^2}$
$= \sqrt{\frac{25}{25}}$
$= 1$
$\begin{pmatrix} -3/5 \\ 0 \\ 4/5 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$

18. $f(x) = (4-3x^2)^{-1/2}$
$f'(x) = -1/2(4-3x^2)^{-3/2} \cdot 6x$
$= -3x(4-3x^2)^{-3/2}$ (D)

19. $(3-x)(2-x)$


$x < -2$
$x > 3$ (C)

20. $A(r) = 2\pi r^2 + 6\pi r$
$A'(r) = 4\pi r + 6\pi$
$A'(2) = 8\pi + 6\pi$
$= 14\pi$ (C)

21. on x axis, $y = 0$
$6x - 0 + 18 = 0$
$6x = +18$
$x = -3$ P(-3, 0)

b) $m = \frac{-2-6}{8-4} = \underline{-2}$

If L, $m_1 m_2 = -1$
$-2 \times (1/2) = -1$
$y - 0 = 1/2(x+3)$
$y = \frac{1}{2}x + 3/2$

c) $y - 6 = -2(x-4)$
$y - 6 = -2x + 8$
$y = -2x + 14$
$\frac{1}{2}x + \frac{3}{2} = -2x + 14$
$x + 3 = -4x + 28$
$5x = 25$
$x = 5$
$2y = 5 + 3$
$y = 4$ (5, 4).

22.

$\vec{DE} = \underline{e} - \underline{d} = \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix} = 3\begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

$\vec{EF} = \underline{f} - \underline{e} = \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

$\vec{DE} = 3\vec{EF}$ ∴ parallel.

Since E is common pt, D, E, F are collinear.

3 : 1

b) $\vec{DE} \cdot \vec{GE} = 0$ if k.

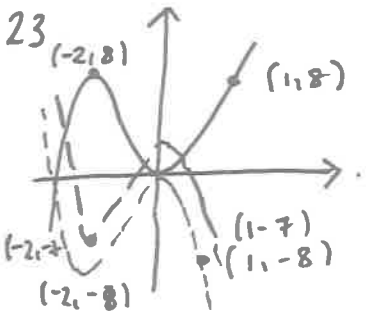
$-9(1-k) + (-18) + (-36) = 0$

$-9 + 9k - 54 = 0$

$9k = 63$

$k = 7$

23



24a)

$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin(A+B) + \sin(A-B)$

$\sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B$

c) $\frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{4\pi}{12} - \frac{3\pi}{12}$

$= \frac{\pi}{12}$.

$2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{2\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

2009 Paper 2

1. $3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

$y = -1^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$ $\quad (-1, 17)$

$y = 3^3 - 3(3^2) - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$ $\quad (3, -15)$

	→ −1	− 7 3 →
$(x-3)$	+	0 − 0 +
$(x+1)$	/	− \ − /

max at $(-1, 17)$ min at $(3, -15)$

2. $f(g(x)) = f(x^2 - 2)$

$= 3(x^2-2) + 1$

$= 3x^2 - 6 + 1$

$= 3x^2 - 5$

$g(f(x)) = g(3x+1)$

$= (3x+1)^2 - 2$

$= 9x^2 + 6x + 1 - 2$

$= 9x^2 + 6x - 1$

b) $6x = 18x + 6$

$-12x = 6 \quad x = -\frac{1}{2}$

3a)

1	1	8	11	−20
		1	9	20
	1	9	20	0 ∴ factor

$(x-1)(x^2+9x+20)=0$

$(x-1)(x+4)(x+5)=0$

b) $\log_2(x+3)+\log_2(x^2+5x-4)=3$

$\log_2(x^3+5x^2-4x+3x^2+15x-12)=3$

$x^3+8x^2+11x-12 = 2^3$

$x^3+8x^2+11x-20=0$

$(x-1)(x+4)(x+5)=0$

$x=1$ ~~$x=-4$~~ ~~$x=-5$~~

4a) $(x+1)^2+(y-2)^2$ at $(5,10)$

$= 6^2+8^2$

$= 100$

∴ $(5,10)$ satisfies equation ∴ lies on circle.

b) Centre $(-1,2)$ Q $(-7,-6)$

$\frac{2-(-6)}{-1-(-7)} = \frac{8}{6} = \frac{4}{3}$

mtgt $= -3/4$

$y+6 = -\frac{3}{4}(x+7)$

$4y+24 = -3x-21$

$4y = -3x-45$

$y = -\frac{3x}{4} - \frac{45}{4}$

c) Centre $(5,10)$

$r = \sqrt{16^2+12^2}$

$= \sqrt{400}$

$= 20$

$(x-5)^2+(x-10)^2 = 400$

C_3: Centre $(-19,-22)$

$r = 20$

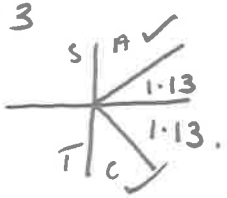
$(x+19)^2+(x+22)^2 = 400.$

5a) $g(x) = 3\cos(2x)$

b) $3\cos 2x = -4\cos 2x + 3$

$7\cos 2x = 3$

$\cos 2x = 3/7$



$2x = 1.13, 5.16$

$x = 0.565, 2.58$ radians

$y = 3\cos(2(0.565))$

$= 1.28$ $(0.6, 1.3)$

$y = 3\cos(2\times 2.58)$

$= 1.3$ $(2.6, 1.3)$

c) $\int_{0.6}^{2.6} -4\cos 2x + 3 - 3\cos 2x \, dx$

$\int_{0.6}^{2.6} -7\cos 2x + 3 \, dx$

$\left[\frac{1}{2}(-7\sin 2x) + 3x\right]_{0.6}^{2.6}$

$= \left[-3.5\sin(5.2) + 7.8\right] - \left[-3.5\sin(1.2) + 1.8\right]$

$= 10.89 + 1.46$

$= 12.35$

6. $N = 61e^{(0.016\times 14)}$

$= 76.3$ million

b) $5.1e^{0.0043t} = 10.2$

$e^{0.0043t} = \frac{10.2}{5.1} = 2$

$0.0043t = \ln 2$

$t = \frac{\ln 2}{0.0043} = 161.2$ years

7a $\underline{p} \cdot (\underline{q} + \underline{r})$

$\underline{p} \cdot \underline{q} + \underline{p} \cdot \underline{r}$

$|\underline{p}|\,|\underline{q}| \cos 30 + |\underline{p}|\,|\underline{r}| \cos 90$

$12 \cos 30 + 4|\underline{r}| \cos 90$

$= 12\left(\frac{\sqrt{3}}{2}\right)$

$= \underline{\underline{6\sqrt{3}}}$

$\underline{r} \cdot (\underline{p} - \underline{q})$

$\underline{r} \cdot \underline{p} - \underline{r} \cdot \underline{q}$

$|\underline{r}|\,|\underline{p}| \cos\theta - |\underline{r}|\,|\underline{q}| \cos\theta$

$|r| \, 4 \cos 90 - |r| \, 3 \cos 120$

$-3|\underline{r}|\left(\frac{-1}{2}\right)$

$= \underline{\underline{\frac{3}{2}|\underline{r}|}}$