

Perth Academy



Mathematics

Higher

2006

Paper 2

Calculator

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

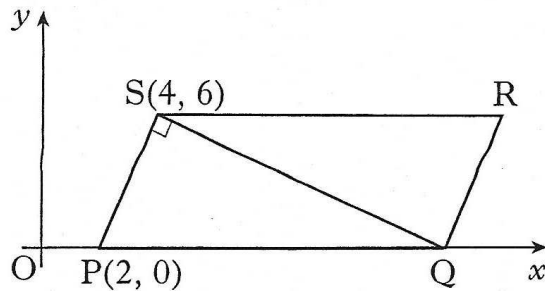
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

1. PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x -axis, as shown.

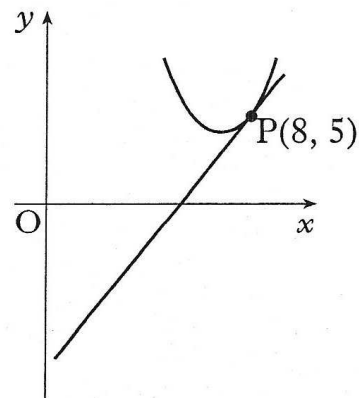
The diagonal QS is perpendicular to the side PS.



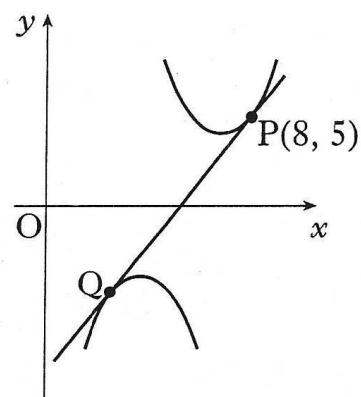
- (a) Show that the equation of QS is $x + 3y = 22$. 4
- (b) Hence find the coordinates of Q and R. 2
2. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. 4

3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

(a) Find the equation of this tangent. 4



(b) Show that the tangent found in (a) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q.



4. The circles with equations $(x - 3)^2 + (y - 4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre.

Determine the radius of the larger circle. 5

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x . 4

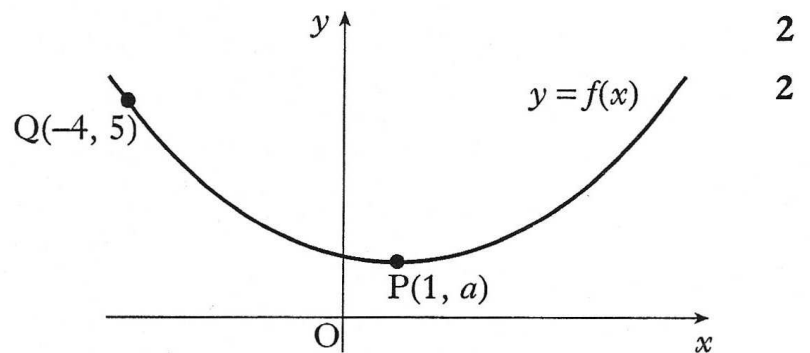
6. P is the point $(-1, 2, -1)$ and Q is $(3, 2, -4)$.
- (a) Write down \vec{PQ} in component form. 1
- (b) Calculate the length of \vec{PQ} . 1
- (c) Find the components of a unit vector which is parallel to \vec{PQ} . 1

7. The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of:

(a) $y = f(x - 4)$;

(b) $y = 2 + f(x - 4)$.

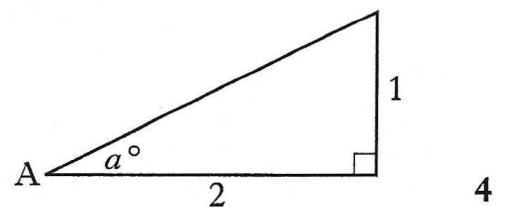


8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of a° at A.

(a) Find the exact values of:

(i) $\sin a^\circ$;

(ii) $\sin 2a^\circ$.



- (b) By expressing $\sin 3a^\circ$ as $\sin(2a + a)^\circ$, find the exact value of $\sin 3a^\circ$. 4

9. If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. 4

10. A curve has equation $y = 7\sin x - 24\cos x$.

(a) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$ 4

- (b) Hence find, in the interval $0 \leq x \leq \pi$, the x -coordinate of the point on the curve where the gradient is 1. 3

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years.

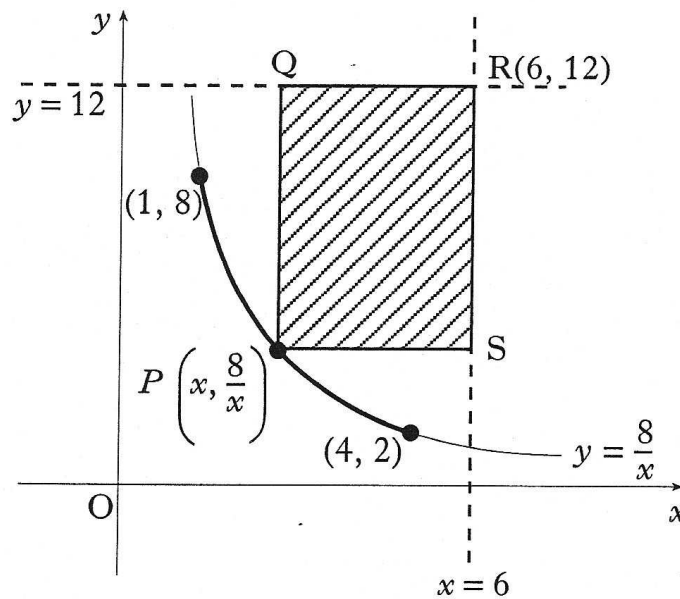
For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.

Is the claim true?

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12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between $(1, 8)$ and $(4, 2)$
- R is the point $(6, 12)$.



- (a) (i) Express the lengths of PS and RS in terms of x , the x -coordinate of P.
(ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}$$

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- (b) Find the greatest and least possible values of A and the corresponding values of x for which they occur.

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