

Perth Academy



Mathematics

Higher

2006

Paper 1

Non-Calculator

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

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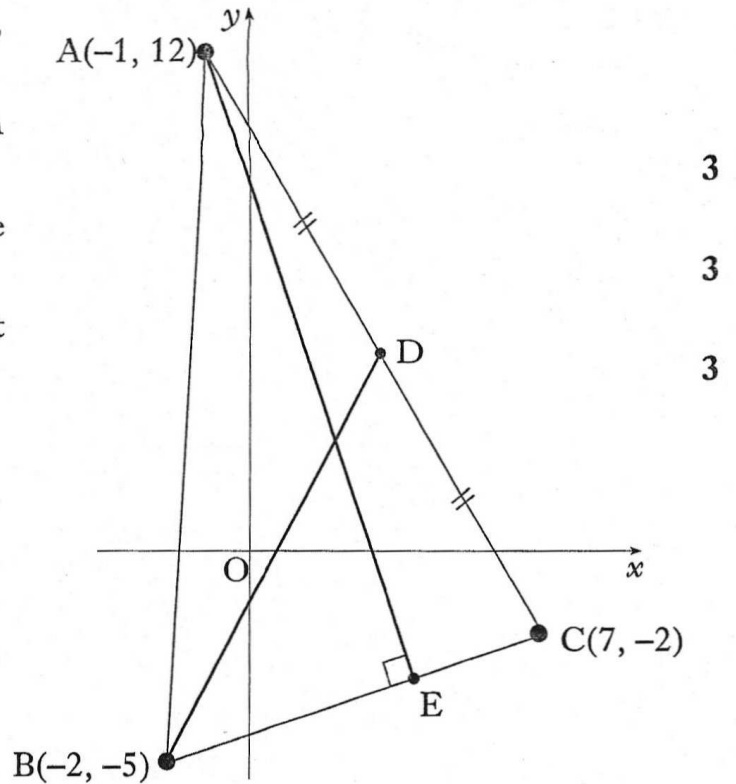
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

1. Triangle ABC has vertices $A(-1, 12)$, $B(-2, -5)$ and $C(7, -2)$.

(a) Find the equation of the median BD.

(b) Find the equation of the altitude AE.

(c) Find the coordinates of the point of intersection of BD and AE.



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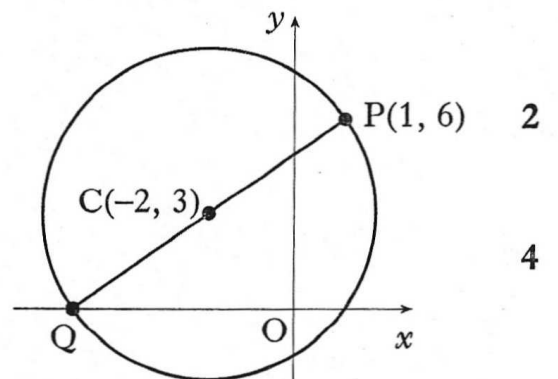
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2. A circle has centre $C(-2, 3)$ and passes through $P(1, 6)$.

(a) Find the equation of the circle.

(b) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.



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3. Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

(a) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$.

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(b) Determine the least possible value of the product $f(g(x)) \times g(f(x))$.

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4. A sequence is defined by the recurrence relation $u_{n+1} = 0.8u_n + 12$, $u_0 = 4$.

(a) State why this sequence has a limit.

1

(b) Find this limit.

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5. A function f is defined by $f(x) = (2x - 1)^5$.

Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature.

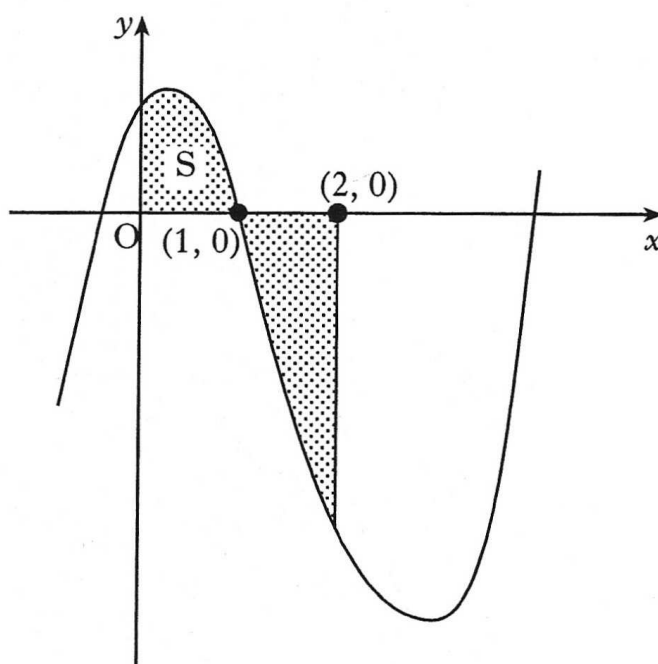
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6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x -axis, the y -axis and the line $x = 2$.

(a) Calculate the shaded area labelled S.

(b) Hence find the total shaded area.



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7. Solve the equation $\sin x^\circ - \sin 2x^\circ = 0$ in the interval $0 \leq x \leq 360$.

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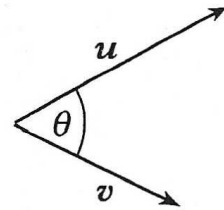
8. (a) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$.

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(b) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$.

1

9. u and v are vectors given by $u = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



- (a) If $u \cdot v = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. 2
- (b) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. 5
- (c) Deduce the only possible value of k . 1
- (d) The angle between u and v is θ . Find the exact value of $\cos \theta$. 3
10. Two variables, x and y , are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a . 4

