Perth Academy

Mathematics

Higher

2005

Paper 2

Calculator

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

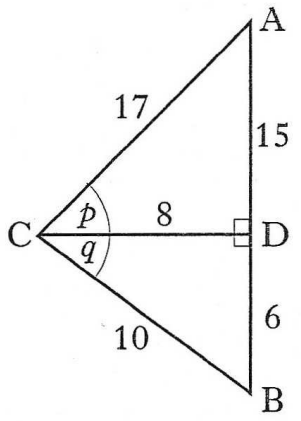
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

1. Find $\int \frac{4x^3 - 1}{x^2}\,dx, \quad x \neq 0.$ 4

2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.



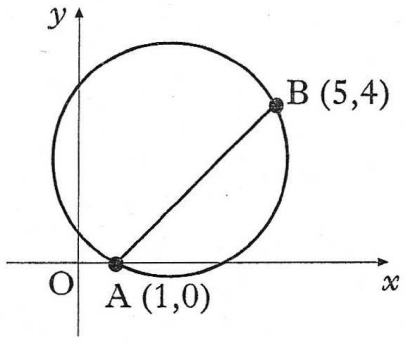
(*a*) Show that the exact value of $\sin(p + q)$ is $\frac{84}{85}$. 4

(*b*) Calculate the exact values of:

(i) $\cos(p + q)$;

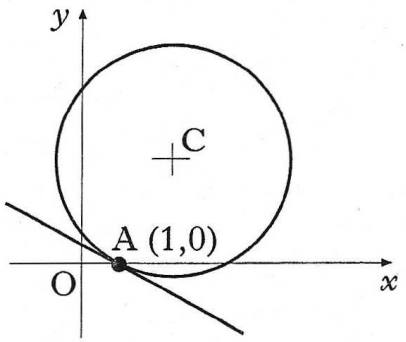
(ii) $\tan(p + q)$. 3

3. (*a*) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.



Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$. 4

(*b*) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$.



Find the equation of the radius CA. 4

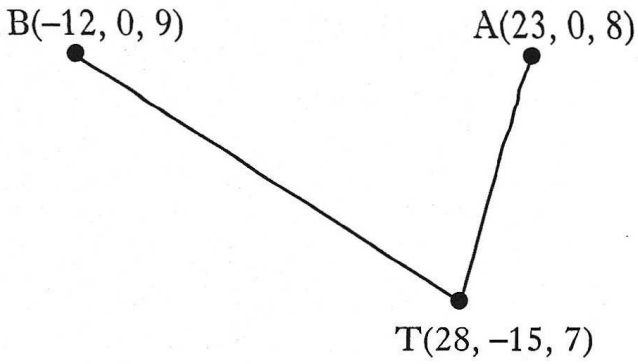
(*c*) (i) Determine the coordinates of the point C.

(ii) Find the equation of the circle. 4

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(–12, 0, 9) and T(28, –15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

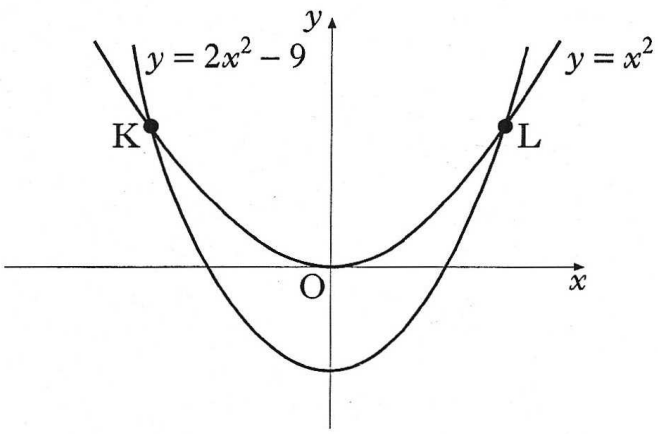


(*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. **2**

(*b*) Calculate the angle between these two beams. **5**

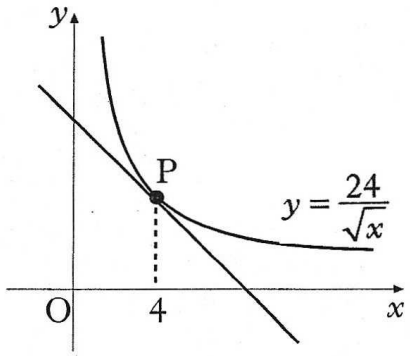
5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves. **8**



6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$. **6**

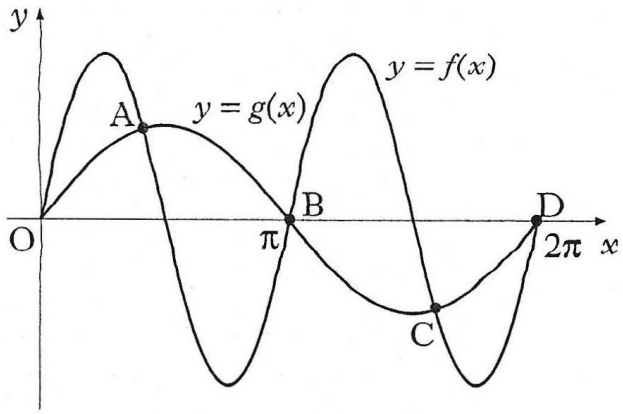


7. Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$. **4**

8. Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \frac{1}{2k}$.



5

9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0{\cdot}06335t}$.

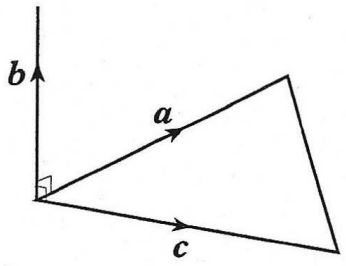
(*a*) What was its value when launched? 1

(*b*) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? 4

10. Vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector $\boldsymbol{b}$ is 2 units long and $\boldsymbol{b}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{c}$.

Evaluate the scalar product $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$.



4

11. (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. 1

(*b*) Hence find the range of values of p for which all the roots of the cubic equation are real. 7