Higher Maths 2005

Paper 2 Solutions

1. $$\int \frac{4x^3 - 1}{x^2} x$$

$$= \int \left(\frac{4x^3}{x^2} - \frac{1}{x^2} \right) dx$$

$$= \int \left(4x - x^{-2} \right) dx$$

$$= \frac{4x^2}{2} - \frac{x^{-1}}{-1} + c$$

$$= 2x^2 + \frac{1}{x} + c$$

2. (a) $\sin(p+q) = \sin p \cos q + \cos p \sin q$

$$= \frac{15}{17} \cdot \frac{4}{5} + \frac{8}{17} \cdot \frac{3}{5}$$

$$= \frac{60}{85} + \frac{24}{85}$$

$$= \frac{84}{85}$$

(b) $\cos(p+q) = \cos p \cos q - \sin p \sin q$

$$= \frac{8}{17} \cdot \frac{4}{5} - \frac{15}{17} \cdot \frac{3}{5}$$

$$= \frac{32}{85} - \frac{45}{85}$$

$$= -\frac{13}{85}$$

(c) $\tan(p+q) = \dfrac{\sin(p+q)}{\cos(p+q)}$

$$= \frac{\frac{84}{85}}{-\frac{13}{85}}$$

$$= \frac{84}{85} \times -\frac{85}{13}$$

$$= -\frac{84}{13}$$

3. (a) $m_{AB} = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

$= \dfrac{(4-0)}{(5-1)}$

$= \dfrac{4}{4}$

$= 1$

$\Rightarrow m_{chord} = -1$ as $m_1.m_2 = -1$ for $\perp$ lines

midpoint = (3,2)

$y - b = m(x - a)$

$y - 2 = -1(x - 3)$

$y - 2 = -x + 3$

$x + y = 5$

(b) $m_T = -\dfrac{1}{3}$

$\Rightarrow m_r = 3$

$y - b = m(x - a)$

$y - 0 = 3(x - 1)$

$y = 3x - 3$

(c) Perpendicular bisector of AB (answer (a))

and radius CA will intersect at centre, C.

$\Rightarrow x + (3x - 3) = 5$

$4x = 8$

$x = 2$

$\Rightarrow y = 3$

$\Rightarrow$ C(2,3)

$r = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}$

$r = \sqrt{(2-1)^2 + (3-0)^2}$

$r = \sqrt{(1)^2 + (3)^2}$

$r = \sqrt{10}$

$\Rightarrow$ Equation of circle: $(x - 2)^2 + (y - 3)^2 = 10$

4. (a) $\overrightarrow{TA} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$ $\quad \overrightarrow{TB} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$

(b) $|\overrightarrow{TA}| = \sqrt{(-5)^2 + (15)^2 + (1)^2}$

$= \sqrt{251}$

$|\overrightarrow{TB}| = \sqrt{(-40)^2 + (15)^2 + (2)^2}$

$= \sqrt{1829}$

$\cos A = \dfrac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$

$\cos A = \dfrac{(-5)(-40) + (15)(15) + (1)(2)}{\sqrt{251}.\sqrt{1829}}$

$\cos A = 0.631$

$A = 50.9°$

5. For points of intersection $y = y$

$\Rightarrow \quad 2x^2 - 9 = x^2$

$x^2 - 9 = 0$

$(x + 3)(x - 3) = 0$

$\Rightarrow \quad x = -3, 3$

$A = \int_{-3}^{3} (x^2 - (2x^2 - 9))dx$

$= \int_{-3}^{3} (9 - x^2)dx$

$= \left[9x - \dfrac{x^3}{3} \right]_{-3}^{3}$

$= \left(9(3) - \dfrac{(3)^3}{3} \right) - \left(9(-3) - \dfrac{(-3)^3}{3} \right)$

$= (27 - 9) - (-27 + 9)$

$= 18 - (-18)$

$= 36 \text{ units}^2$

6. $y = \dfrac{24}{\sqrt{x}}$ when $x = 4$

$y = 24x^{-1/2}$

$\dfrac{dy}{dx} = -12x^{-3/2}$

$\dfrac{dy}{dx} = \dfrac{-12}{\sqrt{(x)^3}}$

$y = \dfrac{24}{\sqrt{(4)}}$

$y = 12$

$\dfrac{dy}{dx} = \dfrac{-12}{\sqrt{(4)^3}}$

$\dfrac{dy}{dx} = -\dfrac{12}{8}$

$\dfrac{dy}{dx} = -\dfrac{3}{2}$

$\Rightarrow$ P(4,12) $m = -\dfrac{3}{2}$

$y - b = m(x - a)$

$y - 12 = -\dfrac{3}{2}(x - 4)$

$2y - 24 = -3x + 12$

$3x + 2y = 36$

7. $\log_4(5 - x) - \log_4(3 - x) = 2$

$\log_4 \dfrac{(5-x)}{(3-x)} = 2$

$4^2 = \dfrac{(5-x)}{(3-x)}$

$16 = \dfrac{(5-x)}{(3-x)}$

$16(3 - x) = 5 - x$

$48 - 16x = 5 - x$

$43 = 15x$

$\dfrac{43}{15} = x$

8. For points of intersection y = y

$$\Rightarrow \quad k\sin 2x = \sin x$$

$$k \times 2\sin x\cos x = \sin x$$

$$2k\sin x\cos x - \sin x = 0$$

$$\sin x(2k\cos x - 1) = 0$$

$$\Rightarrow \quad \sin x = 0 \quad \text{or} \quad 2k\cos x - 1 = 0$$

$$x = 0, \pi, 2\pi \qquad 2k\cos x = 1$$

$$\cos x = \frac{1}{2k}$$

$\Rightarrow$ From diagram, $x = 0, \pi, 2\pi$ correspond to points O, B and D respectively.

Hence, at A and C, $\cos x = \dfrac{1}{2k}$.

9. (a) At launch, $t = 0$

$\Rightarrow V = £252$ million

(b)

$$20 = 252e^{-0.06335t}$$

$$\frac{20}{252} = e^{-0.06335t}$$

$$\log_e\left(\frac{20}{252}\right) = \log_e(e^{-0.06335t})$$

$$\log_e\left(\frac{20}{252}\right) = -0.06335t$$

$$t = \frac{\log_e\left(\frac{20}{252}\right)}{-0.06335}$$

$$t = 40 \text{ years}$$

10. $\boldsymbol{a}\cdot(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$

$= \boldsymbol{a}\cdot\boldsymbol{a} + \boldsymbol{a}\cdot\boldsymbol{b} + \boldsymbol{a}\cdot\boldsymbol{c}$

$= \boldsymbol{a}^2 + |\boldsymbol{a}|\,|\boldsymbol{b}|\cos 90° + |\boldsymbol{a}|\,|\boldsymbol{c}|\cos 60°$

$= (3)^2 + 0 + (3)(3)\cos 60°$

$= 9 + 9.\frac{1}{2}$

$= 13.5$

11. (a)

-1	1	p	p	1
		-1	$1-p$	-1
	1	$p-1$	1	0

$\Rightarrow x = -1$ is a solution

(b) Quotient: $x^2 - (p-1)x + 1$

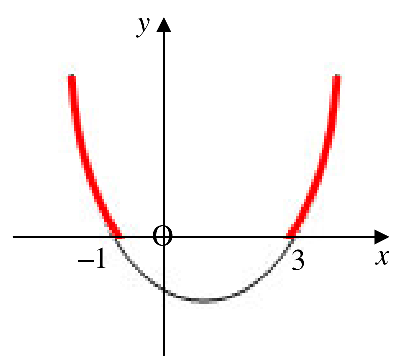
For real roots $b^2 - 4ac \geq 0$

$(p-1)^2 - 4(1)(1) \geq 0$

$p^2 - 2p + 1 - 4 \geq 0$

$p^2 - 2p - 3 \geq 0$

$(p+1)(p-3) \geq 0$



$\Rightarrow p \leq -1$ and $p \geq 3$ for all roots of the cubic function to be real.