2004 Paper 1

1) $x + 3y = -1$

$2x + 5y = 0$

$2x + 6y = -2$

$2x + 5y = 0$

$y = -2$

$x + 3(-2) = -1$

$x = 5$

$(5, -2)$

$m_{AB} = \frac{-2-4}{5-7} = \frac{-6}{-2} = 3$

b) If $\perp$, $m_1 m_2 = -1$

$3 \times (-1/3) = -1$

$y + 2 = -\frac{1}{3}(x-5)$

$3y + 6 = -x + 5$

$3y = -x - 1$

$y = -\frac{1}{3}x - \frac{1}{3}$

2a)
$$\begin{array}{r|rrrr} -1 & 1 & -1 & -5 & -3 \\ & & -1 & 2 & 3 \\ \hline & 1 & -2 & -3 & \boxed{0} \end{array}$$
$\therefore$ factor

$(x+1)(x^2 - 2x - 3)$

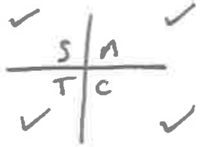
$(x+1)(x+1)(x-3)$

$(x+1)^2(x-3)$

$(-1, 0)$

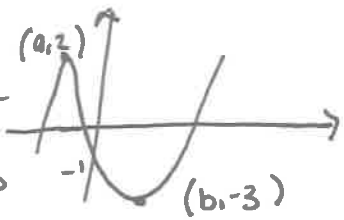
3) $\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}.$



4) $-g(x)$ make y -ve

3 - slide up 3



5a) $\vec{AB} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$= 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$\vec{AB} = 2\vec{BC}$

$\vec{AB}$ and $\vec{BC}$ are //. Since B is common pt, A, B & C are collinear.

b) $\vec{AD} = 4\vec{AB} = \begin{pmatrix} 8 \\ 16 \\ -16 \end{pmatrix} = \underline{d} - \underline{a}$

$\underline{d} = \begin{pmatrix} 5 \\ 20 \\ -9 \end{pmatrix}$ D $(5, 20, -9)$

6. $y = 3\sin x + \cos 2x$

$\frac{dy}{dx} = 3\cos x - 2\sin 2x$

7) $\int_0^2 (4x+1)^{1/2}\,dx$

$= \left[\frac{(4x+1)^{3/2}}{3/2 \times 4}\right]_0^2$

$= \left[\frac{\sqrt{(4x+1)}^3}{6}\right]_0^2$

$= \left[\frac{\sqrt{9}^3}{6}\right] - \left[\frac{\sqrt{1}^3}{6}\right]$

$= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$

8) $(x-5)^2 + 2$

b) $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$

$g'(x) = x^2 - 10x + 27$

$= (x-5)^2 + 2$

Since $(x-5)^2 > 0$, function is always increasing. $(g'(x)) = +$ve.

9) $\log_2(x+1) - 2\log_2(3) = 3$

$\log_2(x+1) - \log_2 3^2 = 3$

$\log_2\left(\frac{x+1}{9}\right) = 3$

$\frac{x+1}{9} = 2^3$

$\frac{x+1}{9} = 8$

$x + 1 = 72 \Rightarrow x = 71$

10) $DEA = 2x + 90$

$\cos DEA = \cos(2x + 90)$

$\cos 2x \cos 90 - \sin 2x \sin 90$

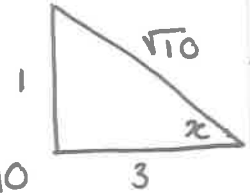
$= 0 - \sin 2x$

$= -2\sin x \cos x$

$= -2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right)$

$= \frac{-6}{10}$

$= \frac{-3}{5}$



$x = \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

11) $y = ax^2 - abx$

at $(2, 0)$

$0 = 4a - 2ab$

at $(1, -6)$

$-6 = a - ab$

$$\begin{aligned} 4a - 2ab &= 0 \\ - \quad 4a - 4ab &= -24 \\ \hline 2ab &= 24 \end{aligned}$$

$\therefore 4a - 24 = 0$

$4a = 24$

$a = 6$

$2ab = 24$

$2(6)b = 24$

$b = 2$

$y = 6x(x - 2)$

b) $y = \int 6x^2 - 12x \, dx$

$= 2x^3 - 6x^2 + C$

$4 = 2(1^3) - 6(1) + C$

$C = 8$

$y = 2x^3 - 6x^2 + 8$

2004 Paper 2

1a) $x - 2y = 0$

$y = \frac{1}{2}x$

$\tan a = \frac{1}{2}$

$a = \tan^{-1}(\frac{1}{2})$

$= 26.6^\circ$

b) $m = \tan(30 + 26.6)$

$= \tan 56.6$

$= 1.5$

2a) $\vec{QP} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$

$|\vec{QP}| = \sqrt{-1^2 + 3^2 + -2^2}$

$= \sqrt{14}$

$\vec{QR} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$

$|\vec{QR}| = \sqrt{-5^2 + 1^2 + 1^2}$

$= \sqrt{27}$

$\cos Q = \frac{\vec{QP} \cdot \vec{QR}}{|\vec{QP}||\vec{QR}|}$

$= \frac{5 + 3 - 2}{\sqrt{14}\sqrt{27}}$

$= \frac{6}{\sqrt{378}}$

$PQR = \cos^{-1}(6 \div \sqrt{378})$

$= 72.0^\circ$

3. $a = 2$

$b = p$

$c = -3$

$b^2 - 4ac$

$p^2 - 4(2)(-3)$

$p^2 + 24$

Since p^2 is always +ve,

$b^2 - 4ac > 0$ ∴ real distinct roots.

4a) If $-1 < k < 1$, limit exists

b) $L = kL + 3$ (If limit = 5)

$5 = 5k + 3$

$2 = 5k$

$\therefore k = \frac{2}{5}$

5a) $y = 6x^2 - x^3$

$\frac{dy}{dx} = 12x - 3x^2 = 12$

$\therefore 3x^2 - 12x + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x = 2$

When $x = 2$ $y = 6(2^2) - (2^3)$

$= 24 - 8$

$= 16$

$(2, 16)$

$m = 12$

$y - 16 = 12(x - 2)$

$y - 16 = 12x - 24$

$y = 12x - 8$

6a)

$3\cos x + 5\sin x = k\cos x\cos a + k\sin x\sin a$

$k\sin a = 5$

$k\cos a = 3$

$\tan a = 5/3$

$a = 59.0^\circ$

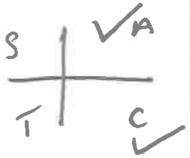
$k = \sqrt{5^2 + 3^2}$

$= \sqrt{34}$

$\therefore \sqrt{34}\cos(x - 59)^\circ$

b) $\sqrt{34}\cos(x-59) = 4$

$\cos(x-59) = \frac{4}{\sqrt{34}}$



$x-59 = 46.7,\ 313.3$

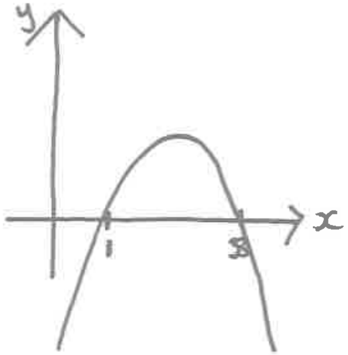
$x = 105.7,\ 372.3$

Since both lie outwith $0<x<90$,

$372.3 - 360 = 12.3$

$\underline{\underline{x = 12.3^\circ}}$

7. roots at $x=1$ and $x=3$



8. Centre A $(6,1)$ P$(5,-1)$

$m = \frac{1-(-1)}{6-5} = \frac{2}{1} = \underline{\underline{2}}$

If ⊥, $m_1 m_2 = -1$

$2 \times \left(-\frac{1}{2}\right) = 1$

$(5,-1)$

$m = -1/2$

$y+1 = -1/2(x-5)$

$2y+2 = -x+5$

$2y = -x+3$

$\underline{\underline{y = -1/2\,x + 3/2}}$

b) $x = 3-2y$

$(3-2y)^2 + y^2 + 10(3-2y) + 6 = 0$

$9 - 12y + 4y^2 + y^2 + 30 - 20y + 6 = 0$

$5y^2 - 30y + 45 = 0$

$y^2 - 6y + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3$ ∴ 1 pt contact.

or $b^2 - 4ac$

$(36) - 4(1)(9) = 0$

Since $b^2-4ac = 0 \Rightarrow$ tgt.

c) $d_{PQ} = \sqrt{(x_P-x_Q)^2 + (y_P-y_Q)^2}$

$= \sqrt{8^2+4^2}$

$= \sqrt{80}$

$\underline{\underline{= 4\sqrt{5}}}$

9) S.A $= 2(x^2) + 2(xh) + 2(2xh)$

$12 = 2x^2 + 6xh$

$12 - 2x^2 = 6xh$

$h = \frac{12-2x^2}{6x}$

$= \frac{12-2x^2}{6x}$

$V = lbh$

$= 2x(x)\left(\frac{12-2x^2}{6x}\right)$

$V = 2x^2\left(\frac{6-x^2}{3x}\right)$

$= \frac{12x^2 - 2x^4}{3x}$

$= \frac{12x - 2x^3}{3}$

$= \frac{2x(6-x^2)}{3}$

b) $V(x) = 4x - \frac{2}{3}x^3$

$V'(x) = 4 - 2x^2 = 0$ at max/min

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

$\cancel{-\sqrt{2}}$ $\quad \underline{\underline{\sqrt{2}}}$

10) $A_1 = A_0 e^{-0.002t}$

$600 = A_0 e^{-0.002\times 1000}$

$600 = A_0 e^{-2}$

$A_0 = \frac{600}{e^{-2}}$

$\underline{\underline{A_0 = 4433.4\text{mg}}}$

b) $e^{-0.002t} = 0.5$

$$-0.002t = \ln(0.5)$$

$$t = \frac{\ln(0.5)}{-0.002}$$

$$\underline{\underline{t = 346.6 \text{ years.}}}$$

11. $\displaystyle\int_1^3 \left(2x - \tfrac{1}{2}x^2\right) dx$

$$\left[x^2 - \tfrac{1}{6}x^3\right]_1^3$$

$$= \left[3^2 - \tfrac{1}{6}(3^3)\right] - \left[1^2 - \tfrac{1}{6}(1^3)\right]$$

$$= \left[9 - \tfrac{27}{6}\right] - \left[1 - \tfrac{1}{6}\right]$$

$$= 9 - \tfrac{27}{6} - 1 + \tfrac{1}{6}$$

$$= 8 - \tfrac{26}{6}$$

$$= 8 - 4\tfrac{1}{3}$$

$$= \underline{\underline{3\tfrac{2}{3}}}$$

rectangle $= 2 \times 1.5 = 3$

$$3\tfrac{2}{3} - 3 = \underline{\underline{\tfrac{2}{3} \text{ units}^2}}$$