Perth Academy

Mathematics

Higher

2004

Paper 1

Non-Calculator

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

1. The point A has coordinates $(7, 4)$. The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B.

(a) Find the gradient of AB. **3**

(b) Hence show that AB is perpendicular to only one of these two lines. **5**

2. $f(x) = x^3 - x^2 - 5x - 3$.

(a) (i) Show that $(x + 1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

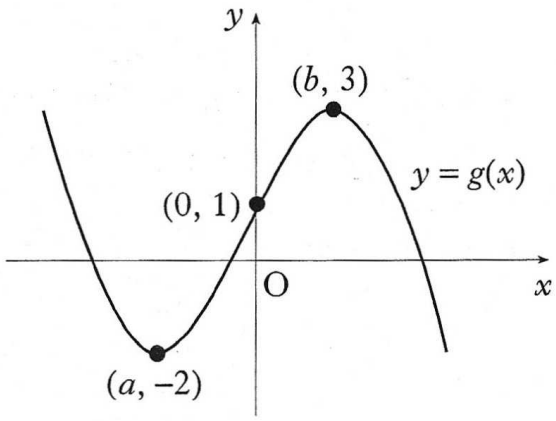
(b) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point. **1**

3. Find all the values of x in the interval $0 \leq x \leq 2\pi$ for which $\tan^2(x) = 3$. **4**

4. The diagram shows the graph of $y = g(x)$.

(a) Sketch the graph of $y = -g(x)$. **2**

(b) On the same diagram, sketch the graph of $y = 3 - g(x)$. **2**



5. A, B and C have coordinates $(-3, 4, 7)$, $(-1, 8, 3)$ and $(0, 10, 1)$ respectively.

(a) Show that A, B and C are collinear. **3**

(b) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$. **2**

6. Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$. **3**

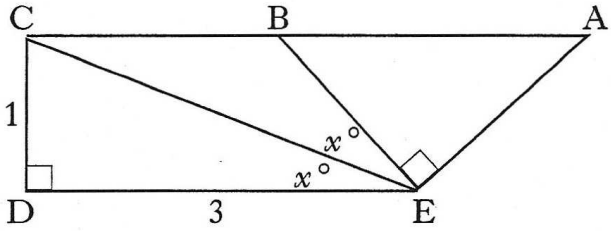
7. Find $\int_0^2 \sqrt{4x+1}\, dx$. **5**

8. (*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$. **2**

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**

9. Solve the equation $\log_2(x + 1) - 2\log_2(3) = 3$. **4**

10. In the diagram
angle DEC = angle CEB = $x°$ and
angle CDE = angle BEA = 90 °.
CD = 1 unit; DE = 3 units.
By writing angle DEA in terms of $x°$, find the exact value of cos(DÊA). **7**



11. The diagram shows a parabola passing through the points (0, 0), (1, −6) and (2, 0).

(*a*) The equation of the parabola is of the form $y = ax(x - b)$.
Find the values of a and b. **3**

(*b*) This parabola is the graph of $y = f'(x)$.
Given that $f(1) = 4$, find the formula for $f(x)$. **5**