# 2003 Paper 1

1) $y = -4x + 1$

If $L_1, m_1 m_2 = -1$

$-4 \times \frac{1}{4} = -1$

$m = \frac{1}{4}$

$(a, b) = (-1, 3)$

$y - 3 = \frac{1}{4}(x+1)$

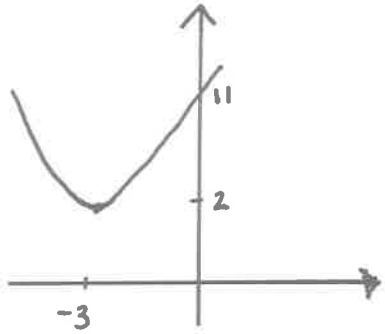
$4y - 12 = x + 1$

$4y = x + 13$

$y = \frac{1}{4}x + \frac{13}{4}$

2a) $(x+3)^2 + 2$

left 3, up 2




at $x = 0$

$y = (3)^2 + 2$

$= 9 + 2$

$= 11$

3) If $L_1 \parallel$ $\underline{u} \cdot \underline{v} = 0$

$\underline{u} \cdot \underline{v} = (3 \times 2) + (2 \times -3) + (0 \times 4)$

$= 6 - 6 + 0$

$= 0 \therefore$ perpendicular.

4) $u_1 = u_0 p + q$

$u_2 = u_1 p + q$

$u_0 = 12$

$u_1 = 15$

$u_2 = 18$

$12p + q = 15$

$-\ 15p + q = 16$

$-3p = -1$

$p = 1/3$

$12(1/3) + q = 15$

$4 + q = 15$

$q = 11$

$u_{n+1} = 1/3\, u_n + 11.$

b) Since $-1 < 1/3 < 1$, limit exists

$L = 1/3\, L + 11$

$2/3\, L = 11$

$2L = 33$

$L = 33/2$

5) $f(x) = \sqrt{x} + \frac{2}{x^2}$

$= x^{1/2} + 2x^{-2}$

$f'(x) = 1/2\, x^{-1/2} - 4x^{-3}$

$= \frac{1}{2\sqrt{x}} - \frac{4}{x^3}$

$f'(4) = \frac{1}{2\sqrt{4}} - \frac{4}{4^3}$

$= \frac{1}{2\sqrt{4}} - \frac{1}{4^2}$

$= \frac{1}{4} - \frac{1}{16}$

$= \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$

6. $\vec{AB} = 1/3\, \vec{AD}$

$3\vec{AB} = \vec{AD}$

$3(\underline{b} - \underline{a}) = \underline{d} - \underline{a}$

$3\underline{b} - 3\underline{a} = \underline{d} - \underline{a}$

$3\underline{b} - 2\underline{a} = \underline{d}$

$\underline{d} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -6 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 8 \\ 3 \\ -1 \end{pmatrix}$

$D(8, 3, -1)$

7) $x^2 + 3x + 4 = 2x + 1$

$x^2 + x + 3 = 0$

If no intersection, $b^2 - 4ac < 0$

$b^2 - 4ac$

$= 1^2 - 4(1)(3)$

$= 1 - 12$

$= -11$

Since $b^2 - 4ac < 0$, line and parabola do not intersect.

8. $\int_0^1 (3x+1)^{-1/2}\, dx = \left[\frac{(3x+1)^{1/2}}{1/2 \times 3}\right]_0^1$

$= \left[\frac{2}{3}\sqrt{(3x+1)^1}\right]_0^1$

$= \left[\frac{2}{3}\sqrt{4}\right] - \left[\frac{2}{3}\sqrt{1}\right]$

$= \frac{4}{3} - \frac{2}{3}$

$= 2/3$ units$^2$

9a) $g(x) = 2x + 3$ $\quad f(x) = \frac{1}{x-4}$

$f(g(x)) = f(2x+3) = \frac{1}{(2x+3) - 4}$

$= \frac{1}{2x-1}$

b) $2x - 1 \neq 0$

$2x \neq 1$

$x \neq 1/2$

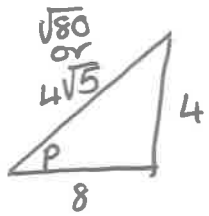
## 2003 Paper 1

10) $x = \sqrt{8^2 + 4^2}$

$= \sqrt{64+16}$

$= \sqrt{80}$

$= 4\sqrt{5}$




$\sin 2p = 2\sin p \cos p$

$= 2\left(\frac{4}{\sqrt{80}}\right)\left(\frac{8}{\sqrt{80}}\right)$

$= \frac{64}{80}$

$= \frac{4}{5}$

$\cos 2p = \cos^2 p - \sin^2 p$

$= \left(\frac{8}{\sqrt{80}}\right)^2 - \left(\frac{4}{\sqrt{80}}\right)^2$

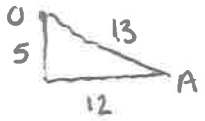
$= \frac{64}{80} - \frac{16}{80}$

$= \frac{48}{80}$

$= 3/5$

$\tan 2p = \frac{\sin 2p}{\cos 2p} = \frac{4/5}{3/5} = \frac{4}{3}$

11. A (12, -5)




$\therefore OA = 13$

Centre A (12, -5)

radius A = 5

Centre B (24, 0)

radius B = AB - 5

= 13 - 5

= 8

$(x-24)^2 + y^2 = 64$.

b) roots at O and B

$x = 0$ $x = 24$

$y = px(x+q)$

$y = px(x-24)$

at (12, -5)

$-5 = p(12)(12-24)$

$-5 = 12p(-12)$

$-5 = -144p$

$p = \frac{-5}{-144} = \frac{5}{144}$

$y = \frac{5}{144}x(x-24)$

12. $3\log_e(2e) - 2\log_e(3e)$

$= \log_e(2e)^3 - \log_e(3e)^2$

$= \log_e 8e^3 - \log_e 9e^2$

$= \log_e 8 + \log_e e^3 - (\log_e 9 + \log_e e^2)$

$= \log_e 8 + 3 - \log_e 9 - 2$

$= \log_e 8 - \log_e 9 + 1$

# 2003 Paper 2

1a)

| | | | | |
|---|---|---|---|---|
| 2 | 6 | −5 | −17 | 6 |
| | 0 | 12 | 14 | −6 |
| | 6 | 7 | −3 | 0 ∴ factor |

$(x-2)(6x^2+7x-3)$

$(x-2)(3x-1)(2x+3)$

2) $y = 4\sin(2x)+1$

3) $x^2+2x-(x^3-x^2-6x)$

$= 2x^2-x^3+8x$

$\int_0^4 2x^2-x^3+8x\,dx$

$\left[\frac{2x^3}{3}-\frac{x^4}{4}+4x^2\right]_0^4$

$= \frac{2(4^3)}{3}-\frac{4^4}{4}+4(4^2)-[0]$

$= \frac{128}{3}-64+64$

$= \frac{128}{3}$ units$^2$

4a) When $x=1$, $y = 1^3+2(1^2)-3(1)+2$

$y = 2$

$(a,b) = (1,2)$

$\frac{dy}{dx} = 3x^2+4x-3$

$m = 4$

$\frac{dy}{dx}$ at $(1,2)$

$y-2 = 4(x-1)$

$\frac{dy}{dx} = 3+4-3$

$y-2 = 4x-4$

$= 4$

$y = 4x-2$

b)

$x^2+(4x-2)^2-12x-10(4x-2)+44=0$

$= x^2+16x^2-16x+4-12x-40x+20+44=0$

$17x^2-68x+68=0$

$17(x^2-4x+4)=0$

$17(x-2)(x-2)=0$

$x=2$

at $x=2$, $y = 4(2)-2$

$= 8-2$

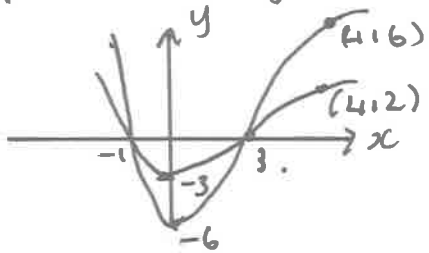
$= 6$

$(2,6)$

5)

$f(-x)$ - reflect in y-axis (change x sign)

$2f(-x)$ - double y-coord




6) $f(x) = \cos 2x - 3\sin 4x$

$f'(x) = -2\sin 2x - 12\cos 4x$

$f'(\pi/6) = -2\sin\left(\frac{\pi}{3}\right) - 12\cos\left(\frac{2\pi}{3}\right)$

$= -2\left(\frac{\sqrt{3}}{2}\right) - 12\left(\frac{-1}{2}\right)$

$= -\sqrt{3}+6$

7) $2\sin x + 5\cos x$

$= k\sin x\cos\alpha + k\cos x\sin\alpha$

$k\sin\alpha = 5$ $\quad k = \sqrt{5^2+2^2}$

$k\cos\alpha = 2$ $\quad = \sqrt{29}$

$\tan\alpha = \frac{5}{2}$

$\alpha = 68.2°$ $\quad y = \sqrt{29}\sin(x+68.2)°$

b) at minimum, $y = -\sqrt{29}$

$\sqrt{29}\sin(x+68.2) = -\sqrt{29}$

$\sin(x+68.2) = -1$

$x+68.2° = 270$

$x = 201.8°$

$P(201.8°, -\sqrt{29})$

# 2003 Paper 2

8a) $A_\Delta = \frac{1}{2}bh = \frac{1}{2}x^2$ or $\frac{x^2}{2}$

$Vol = \frac{1}{2}x^2 \times L$

$216000 = \frac{x^2 L}{2}$

$L = \frac{216000}{x^2}$

$A_\square = x\left(\frac{216000}{x^2}\right) = \frac{216000}{x}$

$A(x) = 2\left(\frac{x^2}{2}\right) + 2\left(\frac{216000}{x}\right)$

$= x^2 + \frac{432000}{x}$

$= x^2 + 432000x^{-1}$

$A'(x) = 2x - 432000x^{-2} = 0$ at max/mn

$0 = 2x - \frac{432000}{x^2}$

$2x = \frac{432000}{x^2}$

$2x^3 = 432000$

$x^3 = 216000$

$x = 60$ cm

| | −7 | 60 | −7 |
|---|---|---|---|
| | − | 0 | + |
| | \ | — | / |

9)

$\underline{a}\cdot(\underline{a}+\underline{b}) = \underline{a}\cdot\underline{a} + \underline{a}\cdot\underline{b}$

$= |\underline{a}||\underline{a}|\cos 0 + |\underline{a}||\underline{b}|\cos\theta$

$= 5\times5\times1 + 5\times4\times\cos\theta$

$= 25 + 20\cos\theta$

$20\cos\theta + 25 = 36$

$20\cos\theta = 11$

$\cos\theta = 11/20$

$\theta = 56.6°$

10) $3\cos 2x + 10\cos x - 1 = 0$

$3(2\cos^2 x - 1) + 10\cos x - 1 = 0$

$6\cos^2 x - 3 + 10\cos x - 1 = 0$

$6\cos^2 x + 10\cos x - 4 = 0$

$2(3\cos^2 x + 5\cos x - 2) = 0$

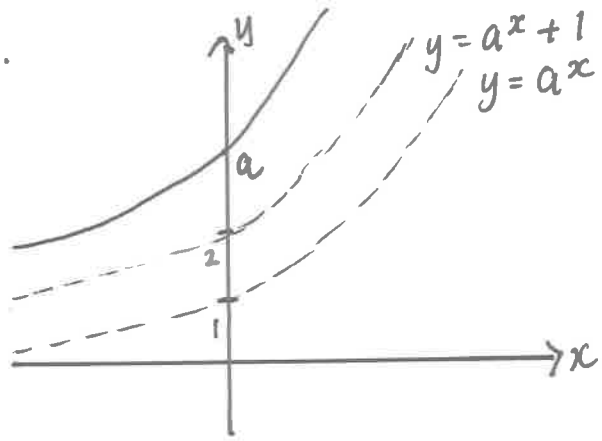
$2(3\cos x - 1)(\cos x + 2) = 0$

$\cos x = 1/3$ ~~$\cos x = -2$~~

$x = 1.23$, ~~5.05~~

when $0 < x < \pi$

11.




$y = a^{x+1}$

$= a^x * a^1$

$= a^x a$

at $x = 0$

$y = a$.

b) $a^x + 1 = a^{x+1}$

$a^x + 1 = a^x \times a$

$1 = a^x \times a - a^x$

$1 = a^x(a-1)$

$a^x = \frac{1}{a-1}$

$\log_a\left(\frac{1}{a-1}\right) = \log_a a^x$

$\log_a\left(\frac{1}{a-1}\right) = x\log_a a$

$x = \log_a\left(\frac{1}{a-1}\right)$