Perth Academy

Mathematics

Higher

2003

Paper 2

Calculator

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\quad \boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\quad \boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

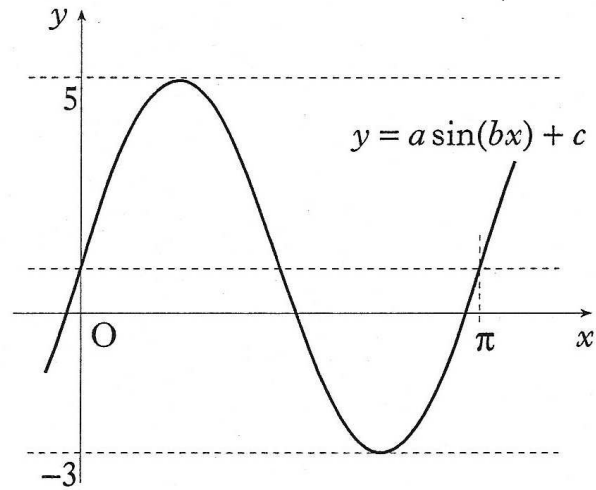1. $f(x) = 6x^3 - 5x^2 - 17x + 6.$

   (a) Show that $(x - 2)$ is a factor of $f(x)$.

   (b) Express $f(x)$ in its fully factorised form. **4**

2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.
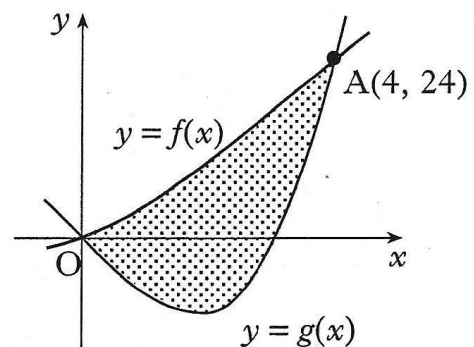
   Determine the values of a, b and c. **3**

3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

   Find the shaded area enclosed between the curves. **5**

4. (a) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

   (b) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**
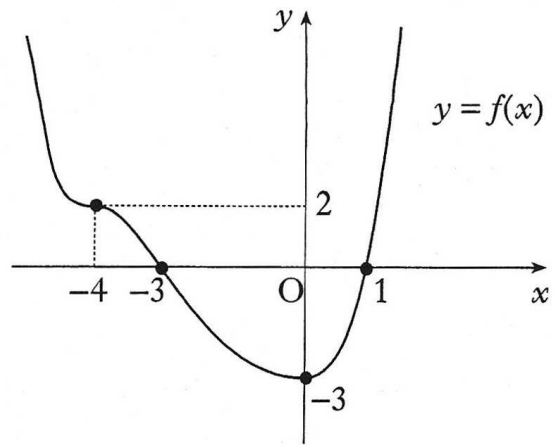
5. The diagram shows the graph of a function f.

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

(a) Sketch the graph of $y = f(-x)$.

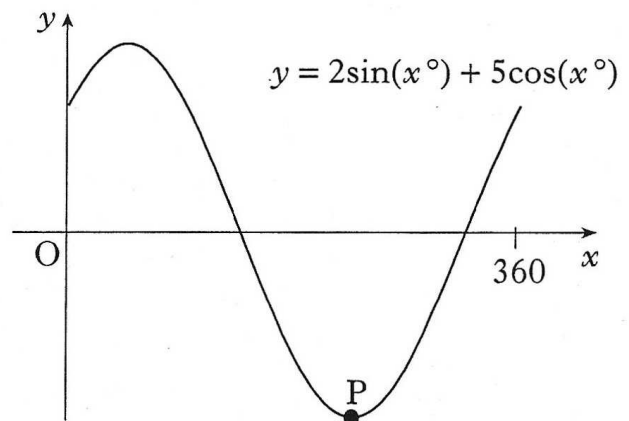(b) On the same diagram, sketch the graph of $y = 2f(-x)$.

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6. If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\dfrac{\pi}{6}\right)$.

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7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.

(a) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a < 360$.
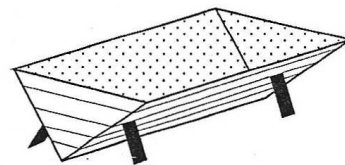
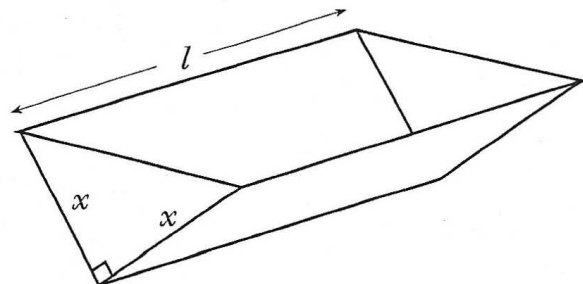(b) Find the coordinates of the minimum turning point P.

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8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.

The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.
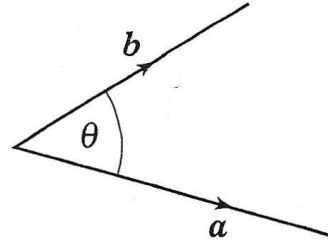
(a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \dfrac{432000}{x}$.

3

(b) Find the value of x which minimises this surface area.

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9. The diagram shows vectors a and b.

   If $|a| = 5$, $|b| = 4$ and $a.(a + b) = 36$, find the size of the acute angle θ between a and b.

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10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places.

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11. (a) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.

    (ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$.

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(b) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\dfrac{1}{a-1}\right)$.

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