Perth Academy

Mathematics

Higher

2003

Paper 1

Non-Calculator

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

1. Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$. **3**

2. (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. **2**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

3. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.
Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**

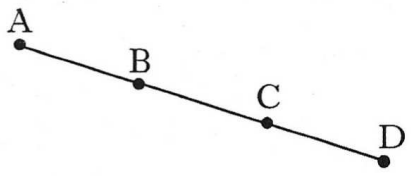
4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

(*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

6. A and B are the points (–1, –3, 2) and (2, –1, 1) respectively.
B and C are the points of trisection of AD, that is AB = BC = CD.
Find the coordinates of D. **3**



7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

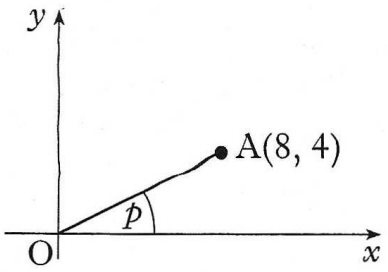
8. Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

9. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

(*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

(*b*) Write down any restriction on the domain of h. **1**

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

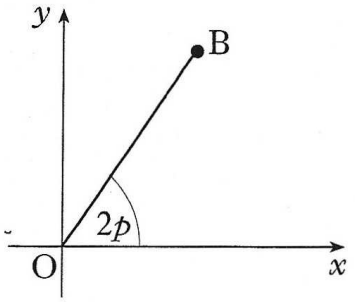


(*a*) Find the exact values of:

(i) $\sin(2p)$;

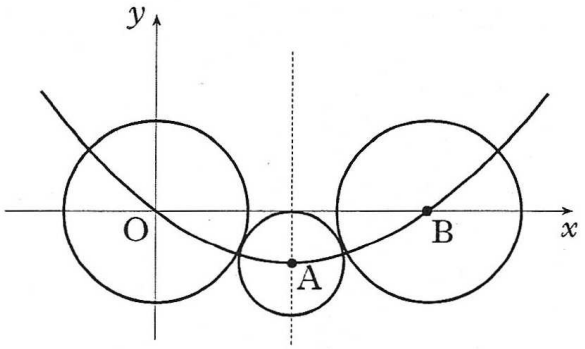
(ii) $\cos(2p)$. **5**

The line OB is inclined at an angle $2p$ radians to the x-axis.



(*b*) Write down the exact value of the gradient of OB. **1**

11.
- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of p and q. **2**

12. Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. **4**