1) $(x+1)^2 + (y-1)^2 = 13$

Centre $= C(-1, 1)$

$M_{CP} = \frac{1-3}{(-1)-2}$

$= \frac{2}{3}$

$\therefore M_{tangent} = -\frac{3}{2}$

$y = mx + c$

$y = -\frac{3}{2}x + c$

$(2, 3)$

$3 = -3 + c$

$c = 6$

$y = -\frac{3}{2}x + 6$

2) $\vec{PR} = \underset{\sim}{r} - \underset{\sim}{p}$

$= \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$

$\vec{PQ} = 2\vec{QR}$

$\Rightarrow (\underset{\sim}{q} - \underset{\sim}{p}) = 2(\underset{\sim}{r} - \underset{\sim}{q})$

$\Rightarrow 3\underset{\sim}{q} = 2\underset{\sim}{r} + \underset{\sim}{p}$

$\Rightarrow 3\underset{\sim}{q} = \begin{pmatrix} 10 \\ 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

$\Rightarrow 3\underset{\sim}{q} = \begin{pmatrix} 9 \\ 3 \\ -6 \end{pmatrix}$

$\Rightarrow \underset{\sim}{q} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

ie Q $(3, 1, -2)$

3) $f(x) = \sin x$, $g(x) = 2x$

(a)(i) $f(g(x)) = f(2x)$

$= \sin(2x)$

(ii) $g(f(x)) = g(\sin x)$

$= 2\sin x$

(b) $2f(g(x)) = g(f(x))$ $\quad 0 \leq x \leq 360$

$\Rightarrow 2\sin 2x = 2\sin x$

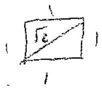
$\Rightarrow 4\sin x\cos x = 2\sin x$

$\Rightarrow 2\sin x(2\cos x - 1) = 0$

$\Rightarrow \sin x = 0$ or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0, 180, 360$ or $x = 60, 300$.

4) $y = 2x^2 - 7x + 10$



$\frac{dy}{dx} = 4x - 7$

$(\tan 45 = 1)$

$4x - 7 = 1$

$x = 2$

$y = 2(2^2) - 7(2) + 10$

$= 4$

$(2, 4)$

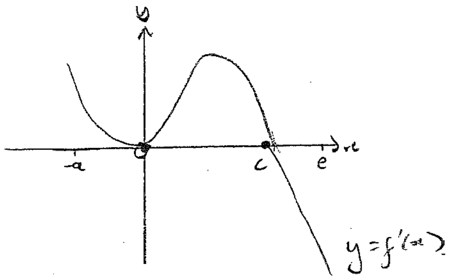
5) $\sin(a+b) = \sin a\cos b + \cos a\sin b$ $\qquad AC = \sqrt{2}$, $BC = \sqrt{10}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{3}{\sqrt{10}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{10}}\right)$

$= \frac{3}{2\sqrt{5}} + \frac{1}{2\sqrt{5}}$

$= \frac{2}{\sqrt{5}}$

6)

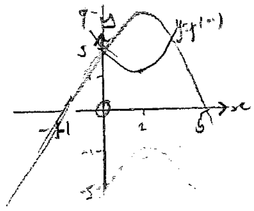


7)(a) $f(x) = x^2 - 4x + 5$

$= (x-2)^2 + 5 - 4$

$= (x-2)^2 + 1$

(b) (i)



$y = 10 - f(x)$

$\Rightarrow y = 5 - x^2 + 4x$

$y = 0$ when $x^2 - 4x - 5 = 0$

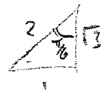
$\Rightarrow (x-5)(x+1) = 0$

$\Rightarrow x = 5, x = -1$.

8)(a) $y = 2\cos 2x$

(b) $y = -\sqrt{3}$

Intersect when: $\cos 2x = -\frac{\sqrt{3}}{2}$



S | A
T | C

$2x = \frac{5\pi}{6}, \frac{7\pi}{6}$

$x = \frac{5\pi}{12}, \frac{7\pi}{12}$

$\therefore B\left(\frac{7\pi}{12}, -\sqrt{3}\right)$

9) (a) $k\sin(x-a) = k\sin x\cos a - k\cos x\sin a$ $\quad (k>0, 0\le a\le 2\pi)$

$= \sin x - \cos x$

$\therefore k\cos a = 1$

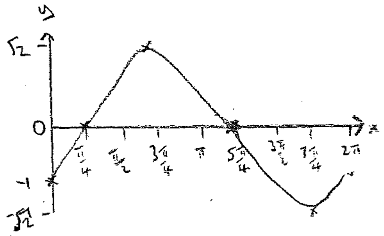
$k\sin a = 1$

$\Rightarrow k = \sqrt{2}$, $\tan a = 1$

$\Rightarrow a = \frac{\pi}{4}$

$\therefore \sqrt{2}\sin(x-\frac{\pi}{4}) = \sin x - \cos x$

(b)



10) (a) $f(x) = (8-x^3)^{\frac{1}{2}}$, $x<2$

$f'(x) = \frac{1}{2}(-3x^2)(8-x^3)^{-\frac{1}{2}}$

$= -\frac{3x^2}{2\sqrt{8-x^3}}$

(b) $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}}\,dx = -\frac{2}{3}(8-x^3)^{\frac{1}{2}} + c$

11) "$y = mx + c$" $\quad m_{AB} = \frac{0-1}{0.5-0} = -2$

$\log_5 y = -2\log_5 x + c$

$\Rightarrow \log_5 y = -2\log_5 x + 1$

$\Rightarrow \log_5 y = \log_5 x^{-2} + 1$

$\Rightarrow \log_5 y = \log_5 x^{-2} + \log_5 5$

$\Rightarrow \log_5 y = \log_5 5x^{-2}$

$\Rightarrow y = 5x^{-2}$

ie $k = 5$, $n = -2$..

1) (a) Let M be point midway between A & B.

Then M is the point:

$\left(\frac{(-1)+(-3)}{2}, \frac{6+(-2)}{2}\right)$

i.e. $M(-2, 2)$

$\therefore$ p: $M(-2,2), C(5,2)$

$y = 2$

(b) q: $\left(\frac{5+(-3)}{2}, \frac{2+(-2)}{2}\right)$

$(1, 0)$

$M_{BC} = \frac{2-(-2)}{5-(-3)}$

$= \frac{1}{2}$

$\therefore M_q = -2$

$y = mx + c$

$y = -2x + c$

$(1, 0)$

$0 = -2 + c$

$c = 2$

$y = 2 - 2x$

(c) At point of intersection:

$2 = 2 - 2x$

$x = 0$

$y = 2$.

$(0, 2)$

2) (a) $B(6,6,0)$

(b) $\vec{DA} = \underline{a} - \underline{d}$

$= \begin{pmatrix}6\\0\\0\end{pmatrix} - \begin{pmatrix}3\\3\\8\end{pmatrix}$

$= \begin{pmatrix}3\\-3\\-8\end{pmatrix}$

$\vec{DB} = \underline{b} - \underline{d}$

$= \begin{pmatrix}6\\6\\0\end{pmatrix} - \begin{pmatrix}3\\3\\8\end{pmatrix}$

$= \begin{pmatrix}3\\3\\-8\end{pmatrix}$

(c) $\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$

$\Rightarrow \cos\theta = \frac{9 + (-9) + 64}{\sqrt{3^2+(-3)^2+(-8)^2}\sqrt{3^2+3^2+(-8)^2}}$

$\Rightarrow \theta = 38.7^\circ$ (1DP)

3) (a) $y = 2x^3 - 7x^2 + 4x + 4$

$\Rightarrow \frac{dy}{dx} = 6x^2 - 14x + 4$

A turning point $\frac{dy}{dx} = 0$.

$\Rightarrow 6x^2 - 14x + 4 = 0$

$\Rightarrow 3x^2 - 7x + 2 = 0$

$\Rightarrow (3x - 1)(x - 2) = 0$

$\Rightarrow x = 2$ or $x = \frac{1}{3}$.

$\therefore$ x-coordinate of maximum is $x = \frac{1}{3}$ (from dgm)

(b) $2x^3 - 7x^2 + 4x + 4$

2	2	-7	4	4
		4	-6	-4
	2	-3	-2	0

$\therefore 2x^3 - 7x^2 + 4x + 4 = (x-2)(2x^2 - 3x - 2)$

$= (x-2)(2x+1)(x-2)$

(c) $A(-\frac{1}{2}, 0)$

$2x^3 - 7x^2 + 4x + 4 < 0$ when $x < -\frac{1}{2}$

4) ↑ 0.5m, 20% trim.

(a) Let u_{n+1} be height at end of year $n+1$.

then $u_{n+1} = 0.5 + 0.8u_n$

$-1 < 0.8 < 1 \therefore \exists$ a limit.

$L = 0.5 + 0.8L$

$\Rightarrow 0.2L = 0.5$

$\Rightarrow L = \frac{0.5}{0.2}$

$\Rightarrow L = \frac{10}{4}$

$= 2.5$m

(b) Let new percentage cut be $\frac{p}{100}$ and $q = 1-p$,

then:

$0.5 + 2q \leq 2$

$2q \leq 1.5$

$q \leq 0.75$

i.e. $p = 25\%$.

5) $y = 1 + 10x - 2x^2$ & $y = 1 + 5x - x^2$

Intersect when:

$1 + 10x - 2x^2 = 1 + 5x - x^2$

$\Rightarrow x^2 - 5x = 0$

$\Rightarrow x(x-5) = 0$

$\Rightarrow x = 0$ or $x = 5$.

$\therefore$ Area $= \int_0^5 \left((1 + 10x - 2x^2) - (1 + 5x - x^2) \right) dx$

$= \int_0^5 (5x - x^2)\, dx$

$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3 \right]_0^5$

$= \left(\frac{5}{2}(5^2) - \frac{1}{3}(5^3) \right) - (0 - 0)$

$= \frac{125}{2} - \frac{125}{3}$

$= \frac{125}{6}$

$= 20\frac{5}{6}$

6) $y = 2\sin(x - \frac{\pi}{6})$ $\quad (x = \frac{\pi}{3})$

$\Rightarrow \frac{dy}{dx} = 2\cos(x - \frac{\pi}{6})$

$\therefore \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{3}} = 2\cos(\frac{\pi}{3} - \frac{\pi}{6})$

$= 2\cos(\frac{\pi}{6})$

$= \sqrt{3}$

$y_{x=\frac{\pi}{3}}$: $y = 2\sin(x - \frac{\pi}{6})$

$y = 2\sin(\frac{\pi}{6})$

$= 1$

tangent: $y = mx + c$

$y = \sqrt{3}x + c$

$(\frac{\pi}{3}, 1)$

$1 = \frac{\sqrt{3}\pi}{3} + c$

$c = 1 - \frac{\sqrt{3}\pi}{3}$

$\therefore y = \sqrt{3}x + 1 - \frac{\sqrt{3}\pi}{3}$

7) $y = \log_3(x-2) + 1$

Intersects x-axis when $y = 0$.

$0 = \log_3(x-2) + 1$

$\Rightarrow \log_3(x-2) = -1$

$\Rightarrow x - 2 = 3^{-1}$

$\Rightarrow x = 2\frac{1}{3}$

8) $a = 2(4-t)^{\frac{1}{2}}$ $\quad 0 \le t \le 4$

Starts at rest: $t = 0, v = 0$.

$a = \frac{dv}{dt}$.

$\therefore v = \int 2(4-t)^{\frac{1}{2}}\, dt$

$= 2(4-t)^{3/2}(-\frac{2}{3}) + c$

$= -\frac{4}{3}(4-t)^{3/2} + c$

$t = 0, v = 0$:

$0 = -\frac{4}{3}(4^{3/2}) + c$

$0 = -\frac{32}{3} + c$

$c = \frac{32}{3}$

$\therefore v = -\frac{4}{3}(4-t)^{3/2} + \frac{32}{3}$

9) $(1-2k)x^2 - 5kx - 2k = 0$

real roots $\Leftrightarrow$ discriminant ≥ 0 $\forall k \in \mathbb{Z}$.
$\forall k \in \mathbb{Z}$

"$b^2 - 4ac$" $= (-5k)^2 - 4(1-2k)(-2k)$

$= 25k^2 + 8k - 16k^2$

$= 9k^2 + 8k$

$9k^2 + 8k \geq 0$

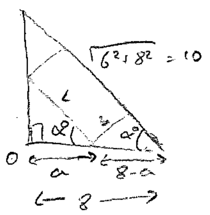
$k(9k + 8) \geq 0$

i.e. $k \geq 0$ or $k \leq -\frac{8}{9}$

$\therefore$ discriminant ≥ 0 $\forall k \in \mathbb{Z}$

(no integers between 0 and $-\frac{8}{9}$).

10) (a) (i)



$\cos\alpha = \frac{a}{l}$ also $\cos\alpha = \frac{8}{10}$

$\therefore \frac{a}{l} = \frac{8}{10}$

$\Rightarrow l = \frac{10a}{8}$

$= \frac{5a}{4}$

(ii) $\sin\alpha = \frac{b}{8-a}$ also $\sin\alpha = \frac{6}{10}$

$\therefore \frac{b}{8-a} = \frac{6}{10}$

$\Rightarrow b = \frac{3}{5}(8-a)$

$\therefore$ Area $= lb$

$= \frac{5a}{4}\left(\frac{3}{5}(8-a)\right)$

$= \frac{3a}{4}(8-a)$

(b) $A(a) = 6a - \frac{3a^2}{4}$

Stationary pt when $A'(a) = 0$:

$6 - \frac{6a}{4} = 0$

$\Rightarrow a = 4.$

check max :

a	3.9	4	4.1
$A'(a)$	-ve	0	+ve
	/	–	\

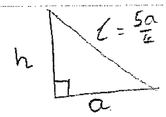
$\therefore$ Max when $a = 4$.

or (a) (i) Scale Factor $= \frac{a}{8}$

$\therefore l = \frac{a}{8} \times 10$

$= \frac{5a}{4}$

(ii)



$h^2 = l^2 - a^2$

$= \left(\frac{5a}{4}\right)^2 - a^2$

$= \frac{25a^2}{16} - a^2$

$= \frac{9a^2}{16}$

$\Rightarrow h = \frac{3a}{16}$

Scale factor $= \frac{8-a}{l}$

$= \frac{4(8-a)}{5a}$

$\therefore b = \frac{4(8-a)}{5a} \cdot h$

$= \frac{\cancel{4}(8-a)}{5\cancel{a}} \cdot \frac{3\cancel{a}}{\cancel{16}4}$

$= \frac{3(8-a)}{5}$

$\therefore$ Area $= bl$

$= \frac{3(8-a)}{5} \cdot \frac{5a}{4}$

$= \frac{3a(8-a)}{4}$

(b) as before.