part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	1.1.8, 1.1.7	CN	C	2001 q1

Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point (2, –1). **3**

Give 1 mark for each •	Illustrations for awarding each •
ans: $2x + 3y = 1$ 3 marks	
•1 ss : express in standard form	•1 $y = -\frac{2}{3}x + \frac{5}{3}$ stated or implied by •2
•2 ic : interpret gradient	•2 $m_{line} = -\frac{2}{3}$ stated or implied by •3
•3 ic : state equation of st line	•3 $y - (-1) = -\frac{2}{3}(x - 2)$

Notes

1 •3 is only available for candidates who attempt to find or state the gradient.

2 •3 is still available even though •1 and •2 may not have been awarded.

example for note 2

$m = 2$

$y - (-1) = 2(x - 2)$ earns •3

example for note 1

$y - (-1) = 7(x - 2)$ earns no marks.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	2.1.7	CN	C	2001 q2

For what value of k does the equation $x^2 - 5x + (k+6) = 0$ have equal roots? **3**

Give 1 mark for each •	Illustrations for awarding each •
ans: $k = \frac{1}{4}$ 3 marks	
•1 ss : know to set disc. to zero	•1 $b^2 - 4ac = 0$ stated or implied by •2
•2 ic : substitute a, b and c into discriminant	•2 $(-5)^2 - 4 \times (k+6)$
•3 pd : process equation in k	•3 $k = \frac{1}{4}$

Notes

1 "= 0" must occur on one of the lines in the solution to this question.

2 If the expression for the discriminant involves x, no marks are available.

3 If the phrase "discriminant = 0" appears at the start, then •1 can only be awarded if •2 is awarded.

ie $disc = 0$

$25 - 4(k+6) = 0$

$k = \frac{1}{4}$

can be awarded 3 marks.

Alternative sol

•1 $x^2 - 5x + k + 6 = 0$

•2 $x^2 - 5x + \left(\frac{5}{2}\right)^2 = 0$ has equal roots

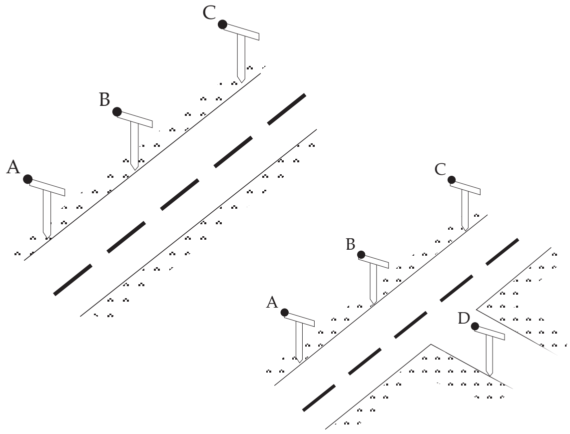
$k + 6 = \frac{25}{4}$

•3 $k = \frac{1}{4}$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	3	3.1.7	CN	C	2001 q3
b)	3	3.1.10	CN	C	

(*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(–8, –10, –2), B (–2, –1, 1) and C(6, 11, 5). Determine whether or not the section of road ABC has been built in a straight line. **3**

(*b*) A further T-rod is placed such that D has coordinates (1, –4, 4). Show that DB is perpendicular to AB. **3**



Give 1 mark for each • | Illustrations for awarding each •

a ans: the road ABC is straight 3 marks

- $\bullet^1$ ic : interpret vector (eg $\overrightarrow{AB}$)
- $\bullet^2$ ic: interpret multiple of vector
- $\bullet^3$ ic : complete proof

$\bullet^1$ *e.g.* $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$

$\bullet^2$ *e.g.* $\overrightarrow{BC} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix} = \frac{4}{3}\overrightarrow{AB}$

$\bullet^3$ a) a common direction exists
and b) a common point exists
so A, B, C collinear

Notes

1 For $\bullet^3$, accept references to "they" are parallel.

2 For (b) Converse of Pythagoras provides an alternative.[See below].

3 Other methods include using the cosine rule and the scalar product.

$\bullet^1$ calculating $AB^2 = 126,\ BD^2 = 27,\ AD^2 = 153$	$\bullet^1$ calculating $AB^2 = 126,\ BD^2 = 27,\ AD^2 = 153$
$\bullet^2$ stating $\cos A\hat{B}D = \frac{126+27-153}{\sqrt{\ldots}\sqrt{\ldots}}$	$\bullet^2$ stating $126 + 27 = 153$
$\bullet^3$ $= 0$ *so* $\angle ABD = 90°$	$\bullet^3$ by converse of Pythagoras $\angle ABD = 90°$

b ans: proof 3 marks

- $\bullet^4$ ic : interpret vector (ie $\overrightarrow{BD}$)
- $\bullet^5$ ss: state requirement for perpend.
- $\bullet^6$ ic : complete proof

$\bullet^4$ $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$\bullet^5$ $\overrightarrow{AB}.\overrightarrow{BD} = 0$

$\bullet^6$ $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9 = 0$

or

$\bullet^4$ $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

$\bullet^5$ $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9$

$\bullet^6$ $= 0$ so AB is at right angles to BD

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	2	1.2.8	NC	C	2001 q4

Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. **2**

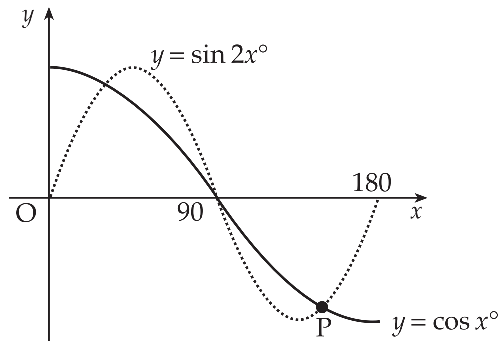
Give 1 mark for each •	Illustrations for awarding each •
ans: $(x+1)^2 - 9$ 2 marks	•[1] $(x+1)^2 \ldots\ldots$
•[1] ss : eg start to complete square	•[2] $(x+1)^2 - 9$
•[2] pd : complete process	***OR***
	•[1] $a = 1$
	•[2] $b = 9$
	OR
	•[1] $x^2 + 2x - 8 \equiv x^2 + 2ax + a^2 - b$
	•[2] $a = 1$ and $b = 9$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	4,1	2.3.5	NC	C	2001 q5

b

5 (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \le x \le 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.
Use your solutions in (*a*) to write down the coordinates of the point P. **1**



Give 1 mark for each • | Illustrations for awarding each •

a ans: 30, 90, 150 — 4 marks

- •[1] ss : use double angle formula
- •[2] pd : factorise
- •[3] pd : process
- •[4] pd : process

•[1] $2\sin x° \cos x°$

•[2] $\cos x°(2\sin x° - 1)$

•[3] $\cos x° = 0, \ \sin x° = \frac{1}{2}$

•[4] $90, 30, 150$

— — — — — — — — —

•[3] $\sin x° = \frac{1}{2}$ and $x = 30, 150$

•[4] $\cos x° = 0$ and $x = 90$

Notes

1 The inclusion of wrong answer(s) means the mark is not awarded (•[4] in method 1, •[3] or •[4] in method 2).

b ans: $\left(150, -\frac{\sqrt{3}}{2}\right)$ — 1 mark

- •[5] ic : interpret graph

•[5] $\left(150, -\frac{\sqrt{3}}{2}\right)$

Notes

2 Accept $y = \cos 150° = -\frac{\sqrt{3}}{2}$ as poor form

3 Wrong formula:

•[1] ✗ $2\cos^2 x° - 1$

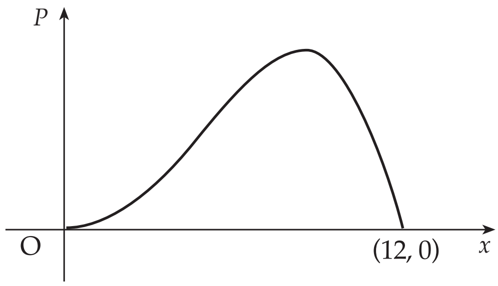
•[2] √ $(2\cos x° + 1)(\cos x° - 1) = 0$

•[3] √ $\cos x° = -\frac{1}{2}, \ \cos x° = 1$

•[4] √ $0, 120, (\not{240}, \not{360})$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	5	1.3.15	NC	C	2001 q6

A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.
Find the value of x which gives the maximum profit.



5

Give 1 mark for each •	Illustrations for awarding each •

ans: x = 9 — 5 marks

- •1 ss : start diff. process
- •2 pd : process
- •3 ss : set derivative to zero
- •4 pc : process
- •5 ic : interpret solutions

- •1 $\frac{dP}{dx} = 36x^2 \ldots\ldots$ ***or*** $\frac{dP}{dx} = \ldots\ldots - 4x^3$
- •2 $\frac{dP}{dx} = 36x^2 - 4x^3$
- •3 $\frac{dP}{dx} = 0$
- •4 $x = 0$ ***and*** $x = 9$
- •5 nature table about $x = 9$ ***and*** $x = 9$

Notes

1 The "= 0" shown in •3 may appear anywhere in the working but must appear explicitly.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	2	1.2.6	NC	C	2001 q7
b)	5	2.3.5	NC	C	

Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for

(i) $f(h(x))$

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$. **5**

Give 1 mark for each •	Illustrations for awarding each •
a ans: $\sin\left(x + \frac{\pi}{4}\right)$, $\cos\left(x + \frac{\pi}{4}\right)$ 2 marks	
•1 ic : interpret composite functions	•1 $\sin\left(x + \frac{\pi}{4}\right)$
•2 ic : interpret composite functions	•2 $\cos\left(x + \frac{\pi}{4}\right)$

Notes

1 One mark may be awarded for "$f\left(x + \frac{\pi}{4}\right)$ ***and*** $g\left(x + \frac{\pi}{4}\right)$"

2 For $\sin x + \frac{\pi}{4}$ *and* $\cos x + \frac{\pi}{4}$ award 1 mark only, unless there is evidence in part b that they have been expanded correctly, in which case treat as bad form and award 2 marks.

3 Do not penalise the appearance of 45°

Give 1 mark for each •	Illustrations for awarding each •
b ans: proof and $x = \frac{\pi}{4}, \frac{3\pi}{4}$ 5 marks	
•3 ss : expand $\sin\left(x + \frac{\pi}{4}\right)$	•3 $\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$ ***and complete***
•4 ic : interpret	•4 $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$
•5 ic : substitute	•5 $\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) - \left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$
•6 pd : start solving process	•6 $\frac{2}{\sqrt{2}}\sin x$
•7 pd : process	•7 $x = \frac{\pi}{4}, \frac{3\pi}{4}$

Notes

4 If the evidence for •5 has no brackets, •5 can only be awarded if there is evidence further on that brackets has been implied.

5 •7 is only available for answers in radians.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	3	3.3.3, 3.3.4	NC	C	2001 q8

Find x if $4\log_x 6 - 2\log_x 4 = 1$. **3**

Give 1 mark for each •	Illustrations for awarding each •
ans: 81 3 marks	
•[1] pd : use log-to-index rule	•[1] $\log_x 6^4 - \log_x 4^2$
•[2] pd : use log-to-division rule	•[2] $\log_x \frac{6^4}{4^2}$
•[3] ic : interpret base for $\log_x a = 1$ and simplify	•[3] all processing leading to $x = 81$

Further illustrations

•[1] $4\log_x 6 - 4\log_x 2 = 1$

$4(\log_x 6 - \log_x 2) = 1$

•[2] $4\log_x \frac{6}{2} = 1$

$\log_x \frac{6}{2} = \frac{1}{4}$

$x^{\frac{1}{4}} = 3$

•[3] $x = 81$

•[1] $4\log_x 6 - 4\log_x 2 = 1$

$4(\log_x 6 - \log_x 2) = 1$

$4\log_x \frac{6}{2} = 1$

•[2] $\log_x \left(\frac{6}{2}\right)^4 = 1$

•[3] $x = 81$

$2\log_x 6 - \log_x 4 = \frac{1}{2}$

•[1] $\log_x 6^2 - \log_x 4 = \frac{1}{2}$

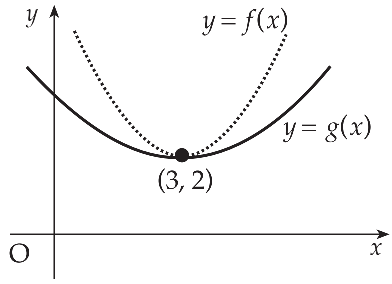
•[2] $\log_x \frac{6^2}{4} = \frac{1}{2}$

$x^{\frac{1}{2}} = 9$

•[3] $x = 81$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	2	1.2.4, 1.3.8	CN	C	2001 q9

The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at (3, 2).
Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$. **2**



Give 1 mark for each • | Illustrations for awarding each •

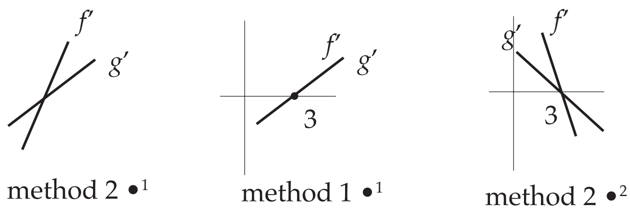
ans: for all k 2 marks

•[1] ss : use $\frac{d}{dx}$("quadratic") = "linear"
•[2] ic : iinterpret stationary point

•[1] st line for f' thr' $(3,0)$, $m_{f'} > 0$
•[2] st line for g' thr' $(3,0)$, $m_{f'} > m_{g'} > 0$

•[1] st lines for f' and g', with $m_{f'} > m_{g'} > 0$
•[2] two lines intersecting at $(3,0)$

Notes
1 Award 0 marks for two curves through anywhere.
2 Further illustrations:

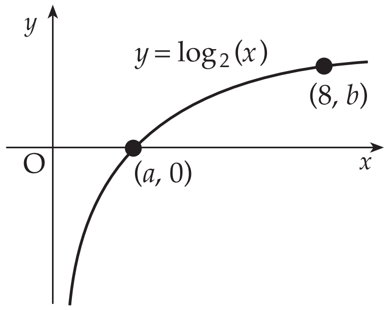


part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	1	1.2.5	CN	B	2001 q10
b)	3	1.2.4, (3.0.0HG?)	CN	A	

10 The diagram shows a sketch of part of the graph of $y = \log_2(x)$.

(*a*) State the values of a and b. **1**

(*b*) Sketch the graph of $y = \log_2(x+1) - 3$. **3**



Give 1 mark for each •

Illustrations for awarding each •

a ans: $a = 1$, $b = 3$ 1 mark

•[1] pd : use $\log_p q = 0 \Rightarrow q = 1$ and evaluate $\log_p p^k$.

•[1] $a = 1$ ***and*** $b = 3$

b ans: sketch 3 marks

•[2] ss : use a translation

•[3] ic : identify one point

•[4] ic : identify a second point

•[2] a "log – shaped" graph of the same orientation

•[3] sketch passes through $(0, -3)$ (labelled)

•[4] sketch passes through $(7, 0)$ (labelled)

Notes

1 Do not penalise any errors made in relation to the asymptote, missing or otherwise.

2 $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ is the correct translation!. You may also consider $\begin{pmatrix} -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ *and* $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$. •[2] is still available and in addition one mark (from •[3] an •[4]) may be awarded for both points consistent with the wrong translation.
Do not consider any other translation.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	4	2.4.5	CN	B	2000 q11
b)	3	2.4.4	CN	B	
c)	3	2.4.4	CN	A	

Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre (–2, –1) and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$. **4**

(ii) Hence show that circles P and Q touch.

(*b*) Find the equation of the tangent to circle Q at the point (–4, 1). **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the *x*-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$. **3**

Give 1 mark for each • | Illustrations for awarding each •

a ans: proof — 4 marks

- •1 ic : interpret centre of circle (P)
- •2 ss: find radius of circle (P)
- •3 ss : find sum of radii
- •4 pd : compare with distance between centres

•1 $r_P = \sqrt{16+25-9} = \sqrt{32} = 4\sqrt{2}$

•2 $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$

•3 $C_P = (4, 5)$

•4 $C_P C_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ ***and*** "so touch"

• •

•1 $C_P = (4, 5)$

•2 $C_P C_Q = \sqrt{6^2 + 6^2} = \sqrt{72}$

•1 $r_P = \sqrt{16+25-9} = \sqrt{32} = 4\sqrt{2}$

•2 $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2} = \sqrt{72}$ ***and*** "so touch"

b ans: $y = x + 5$ — 3 marks

- •5 ss : find gradient of radius
- •6 ss : use $m_1 m_2 = -1$
- •7 ic : state equation of tangent

•5 $m_r = -1$

•6 $m_{tgt} = +1$

•7 $y - 1 = 1(x + 4)$

Note

1 •7 is only available if an attempt has been made to find a perpendicular gradient.

c ans: $x = 2 \pm 2\sqrt{3}$ — 3 marks

- •8 ss : substitute linear into circle
- •9 pd : express in standard form
- •10 pd : solve (quadratic) equation

•8 $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$

•9 $2x^2 - 8x - 16 = 0$

•10 $x = 2 \pm 2\sqrt{3}$

Note

2 •8, •9 and •10 are only available when the answer to part (b) is of the form $y = ax + b$ *where* $a, b \in R,\ a \neq 0$.

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	3	3.1.1	CR	C	2001HG q1
b)	5	3.1.11	CR	C	

The rectangular pyramid A,BCDO has vertices A(3, 2, 5), B(0, 4, 0), C(6, 4, 0), D(6, 0, 0) and the origin O.

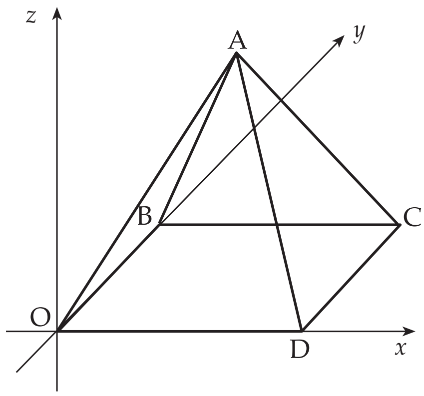
Find

(*a*) (i) $\overrightarrow{DA}$ in component form.

(ii) the coordinates of M, the midpoint of BC.

(iii) $\overrightarrow{DM}$ in component form. **3**

(*b*) Find the size of angle ADM. **5**



Give 1 mark for each • | Illustrations for awarding each •

a **ans:** as shown 3 marks

- •1 ic : interpret info in 3-d sketch
- •2 ic : interpret info in 3-d sketch
- •3 ic : interpret info in 3-d sketch

b **ans:** 56·5° 5 marks

- •4 ss : use $\frac{\overrightarrow{DA}.\overrightarrow{DM}}{|\overrightarrow{DA}|\times|\overrightarrow{DM}|}$
- •5 pd : process $|\overrightarrow{DA}|$
- •6 pd : process $|\overrightarrow{DM}|$
- •7 pd : process scalar product
- •8 pd : find angle

Illustrations:

- •1 $\overrightarrow{DA} = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$
- •2 $M = (3, 4, 0)$
- •3 $\overrightarrow{DM} = \begin{pmatrix} -3 \\ 4 \\ 0 \end{pmatrix}$
- •4 use $\cos A\hat{D}M = \frac{\overrightarrow{DA}.\overrightarrow{DM}}{|\overrightarrow{DA}|\times|\overrightarrow{DM}|}$ stated or implied by work for •8
- •5 $|\overrightarrow{DM}| = \sqrt{25}$
- •6 $|\overrightarrow{DA}| = \sqrt{38}$
- •7 $\overrightarrow{DA}.\overrightarrow{DM} = 17$
- •8 $A\hat{D}M = 56 \cdot 5$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	8	2.1.8, 1.3.9	CN	C	2001HG q2

Show that the line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 - 5x^2 - 2x + 9$ and find the coordinates of the point of contact. **8**

Give 1 mark for each •	Illustrations for awarding each •

ans: proof and (2, 1) 8 marks

- •1 ss : equate functions
- •2 pd : express in standard form (= 0)
- •3 ss : use synthetic division
- •4 pd : find value giving zero remainder
- •5 ic : interpret table for quadratic coeff.
- •6 pd : express cubic as 3 factors
- •7 ic : interpret the equal factors
- •8 ic : interpret third factor

•1 $2x^3 - 5x^2 - 2x + 9 = 2x - 3$

•2 $2x^3 - 5x^2 - 4x + 12 = 0$

•3 *e.g.*

1	2	–5	–4	12

•4

2	2	–5	–4	12
		4	–2	–12
	2	–1	–6	0

•5 $2x^2 - x - 6 = 0$

•6 $(x-2)(x-2)(2x+3)$

•7 equal roots so tangent

•8 pt of contact $= (2,1)$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	1	u/c	CN	C	2001HG q3
b)	4	2.4.4, 1.1.7	CN	C	
c)	4	2.4.4	CN	B	

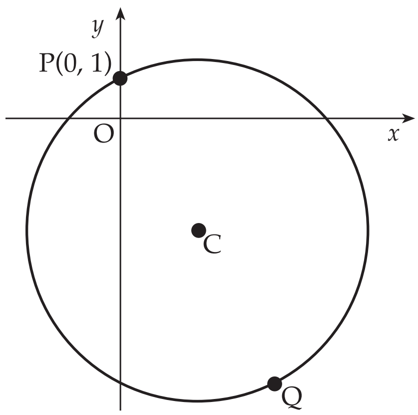
. The diagram shows a circle, centre C, with equation

$(x-2)^2 + (y+3)^2 = 20$

(*a*) Show that the point P(0, 1) lies on this circle. **1**

(*b*) Find the equation of the tangent to this circle at P. **4**

(*c*) If Q is the opposite end of the diameter through P, find the equation of the tangent to this circle at Q. **4**



Give 1 mark for each • | Illustrations for awarding each •

a **ans:** proof — 1 mark

- •[1] pd : substitute coord and complete — •[1] $(0-2)^2 + (1+3)^2 = \ldots 20$

b **ans:** $x - 2y + 2 = 0$ — 4 marks

- •[2] ic : state coord of centre — •[2] $C = (2, -3)$
- •[3] ss : find m_{radius} — •[3] $m_{rad} = -2$
- •[4] ss : use prod of gradients is –1 — •[4] $m_{tgt} = \frac{1}{2}$
- •[5] ic : state equation of tangent. — •[5] $y - 1 = \frac{1}{2}(x - 0)$

c **ans:** $x - 2y - 18 = 0$ — 4 marks

- •[6] ss : method for coord of Q e.g.… — •[6] $\vec{PC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$
- •[7] pd : state coord of Q — •[7] $Q = (4, -7)$
- •[8] ss : use parallel tgts or OW for gradient — •[8] $m_{tgtQ} = \frac{1}{2}$
- •[9] ic : state equation of tangent. — •[9] $y - (-7) = \frac{1}{2}(x - 4)$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	8	1.3.9, 1.1.6	CN	C	2001HG q4
b)	2	1.1.8	CN	B	

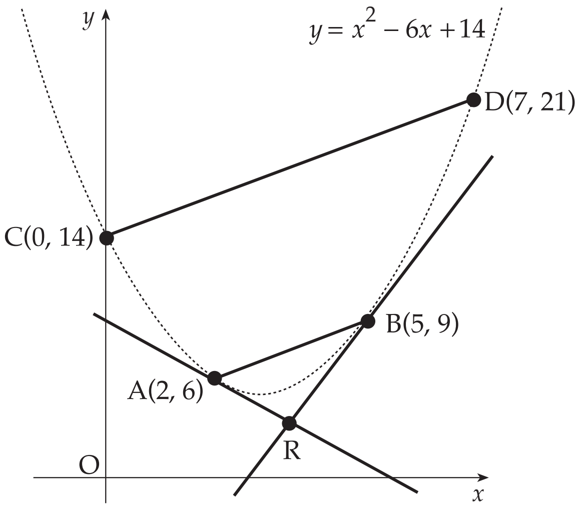
The diagram shows a sketch of a parabola with equation $y = x^2 - 6x + 14$ and two parallel chords AB and CD.

(*a*) (i) Find the equations of the tangents at A(2, 6) and B(5, 9)

(ii) Hence find the coordinates of R, their point of intersection. **8**

(*b*) Let P be the midpoint of AB and Q the midpoint of CD.

Show that P, Q and R are collinear. **2**



Give 1 mark for each • | Illustrations for awarding each •

a **ans: $2x + y - 10 = 0$,**

$4x - y - 11 = 0$, $\left(3\frac{1}{2}, 3\right)$ 8 marks

- •1 ss : use differentiation — •1 $\frac{dy}{dx} = 2x \ \ldots$
- •2 pd : process — •2 $\frac{dy}{dx} = 2x - 6$
- •3 pd : find gradients of tangents — •3 $f'(2) = -2, f'(5) = 4$
- •4 ic : state equation of tangent — •4 $y - 6 = -2(x - 2)$
- •5 ic : state equation of tangent — •5 $y - 9 = 4(x - 5)$
- •6 pd : express equs in standard form — •6 $y = -2x + 10$ ***and*** $y = 4x - 11$
- •7 ss : method for solving — •7 $-2x + 10 = 4x - 11$
- •8 ic : state coord of point of intersection — •8 $R = \left(3\frac{1}{2}, 3\right)$

b **ans: proof** 2 marks

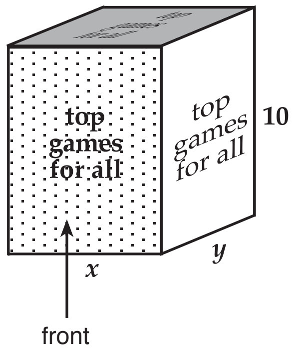
- •9 ic : state coordinates of midpoints — •9 $P = \left(3\frac{1}{2}, 7\frac{1}{2}\right), Q = \left(3\frac{1}{2}, 17\frac{1}{2}\right)$
- •10 ic : interpret y-coords. — •10 P, Q, R lie on line $x = 3\frac{1}{2}$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	3	u/c	CN	A	2001HG q5
b)	7	1.3.15	CN	C	

A games company is using a special cuboid to promote a new 3-D game. The dimensions of the cuboid are x, y and 10 centimetres, as shown, and the volume of the cuboid is 1000 cubic centimetres.

The faces are to be painted in different colours. The cost is as follows:

Faces	Cost
Front and back faces	10p per cm^2
Left and right faces	40p per cm^2
Top and bottom faces	20p per cm^2.



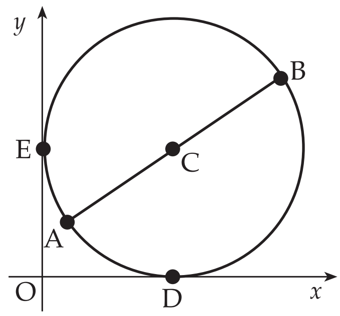
(a) Show that the total cost **in pounds**, C, of the painting is given by $C = 40 + 2x + \dfrac{800}{x}$. **3**

(b) Find the dimensions which will minimise the cost and state this minimum cost. **7**

Give 1 mark for each •		Illustrations for awarding each •
a **ans: proof**	3 marks	
•1 ss : eg start to complete square		•1 $V = 10xy = 1000$
•2 pd : complete process		•2 $C = \frac{1}{100}(2xy \times 20 + 20x \times 10 + 20y \times 40)$
•3 ic : complete proof		•3 $C = \frac{1}{100}\left(200 \times 20 + 200x + 800 \times \frac{100}{x}\right)\ldots$
b **ans: $x = 10$, $y = 5$, Cost = £120**	7 marks	•4 $C = 40 + 2x + 800x^{-1}$
•4 ic : interpret vector (ie $\overrightarrow{BD}$)		•5 $\frac{dC}{dx} = 2\ \ldots$
•5 ss: state requirement for perpend.		•6 $\frac{dC}{dx} = 2 - 800x^{-2}$
•6 ic : complete proof		•7 $2 - \frac{800}{x^2} = 0$
•7 ic : complete proof		•8 $x = 20$
•8 ic : complete proof		•9 $C = 120,\ \ y = 5$
•9 ic : complete proof		•10 nature table about $20^-, 20, 20^+$
•10 ic : complete proof		

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	3	2.4.3	CN	C	2001HG q6
b)	5	2.4.4	CN	C	
c)	3	1.1.9	CN	A	

The circle shown touches the axes at D(5, 0) and E(0, 5). The line through the centre of the circle, with gradient $\frac{3}{4}$, cuts the circle at the points A and B.



(*a*) (i) Find the equation of the circle.

(ii) Find the equation of the line AB. **3**

(*b*) Hence find the coordinates of A and B. **5**

(*c*) A theorem in geometry states that any angle in a semi-circle is a right angle.

Use the points A, B and D to verify this theorem . **3**

Give 1 mark for each •	Illustrations for awarding each •
a **ans:** $(x-5)^2+(y-5)^2=25$	
3x – 4y + 5 = 0 3 marks	
•1 ss : eg start to complete square	•1 $C=(5,5)$
•2 pd : complete process	•2 $(x-5)^2+(y-5)^2=25$
•3 ic : complete proof	•3 $y-5=\frac{3}{4}(x-5)$
b **ans:** A(1, 2), B(9, 8) 5 marks	•4 *start sub e.g.* $y=\frac{3}{4}x+\frac{5}{4}$
•4 ic : interpret vector (ie $\overrightarrow{BD}$)	•5 $\frac{9}{16}x^2-\frac{90}{16}x+\frac{225}{16}$
•5 ss: state requirement for perpend.	
•6 ic : complete proof	•6 $25x^2-250x+225=0$
•7 ic : complete proof	•7 $(x-1)(x-9)=0$ *or equiv.*
•8 ic : complete proof	•8 $A(1,2)$ ***and*** $B(9,8)$
c **ans: Proof** 3 marks	•9 introduce vectors or gradients
•9 ic : complete proof	•10 *e.g.* $m_{DA}=\frac{2}{-4}$, $m_{DB}=\frac{8}{4}$
•10 ic : complete proof	•11 *e.g.* $m_{DA}\times m_{DB}=-1$
•11 ic : complete proof	

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	2	1.3.12	NC	C	2001HG q7
b)	4	1.3.13	NC	C	

The cubic function $f(x) = x^3 + 3x^2 + ax + 5$ has only one stationary point.

(a) Determine the value of a and the nature of the stationary point. **6**

(b) Draw a sketch of the function in the interval $-2 \leq x \leq 1$. **3**

Give 1 mark for each •	Illustrations for awarding each •
a **ans:** a = 3, rising pt of inflexion 6 marks	
•1 ic : interpret composite functions	•1 $f'(x) = 3x^2 \ \ldots$
•2 ic : interpret composite functions	•2 $f'(x) = \ldots + 6x + a$
•3 ss : expand $\sin\left(x + \frac{\pi}{4}\right)$	•3 one st. point $\Rightarrow$ equal roots
•4 ic : interpret	•4 $\Delta = 36 - 12a = 0$ *so* $a = 3$
•5 ic : substitute	•5 $f'(x) = 0 \Rightarrow x = -1$
•6 pd : start solving process	•6 nature table $\Rightarrow$ rising pt of inflexion
b **ans:** sketch 3 marks	
•7 pd : process	•7 st.point at $(-1, 4)$
•8 pd : process	•8 end - points: $(-2, 3)$ *and* $(1, 12)$
•9 pd : process	•9 *sketch*

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	8	2.4.4	CN	B	2001HG q8

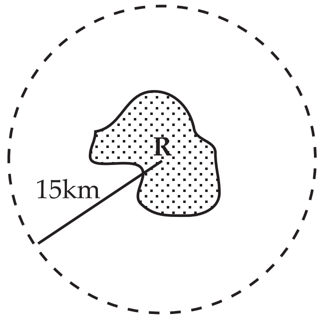
A radar station , situated on an island, has a maximum effective range of 15km.

With respect to suitable axes and a scale where 1 unit represents 1 km,

(i) the radar station is at R(11, –5)

(ii) a ship is sailing along the line whose equation is $y = 5x + 18$.

Will the ship be detected by the radar station?



8

Give 1 mark for each • | Illustrations for awarding each •

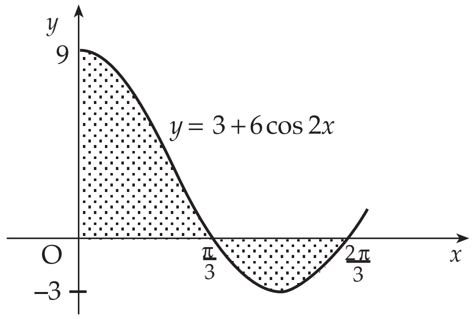
ans: no intersection 8 marks

•1 ss : use equation of circle	•1 ***use*** equ of circle
•2 ic : interpret centre, radius	•2 $(x-11)^2 + (y+5)^2 = 15^2$
•3 ss : know to set up two equations	•3 ***use*** two equations
•4 ic : substitute	•4 $(x-11)^2 + (5x+23)^2 = 15^2$
•5 pd : process	•5 $x^2 - 22x + 121 + 25x^2 + 230x + 529 = 225$
•6 pd : standard form for quadratic	•6 $26x^2 + 208x + 425 = 0$
•7 ss : us discriminant	•7 $\Delta = 208^2 - 4.26.425$
•8 pd : process	•8 –936 so no intersection

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	7	2.2.7	NC	A	2001HG q9

The diagram shows part of the graph of $y = 3 + 6\cos 2x$.

Show that the total shaded area is $\frac{9}{2}\sqrt{3}$ units^2. **7**



Give 1 mark for each • | Illustrations for awarding each •

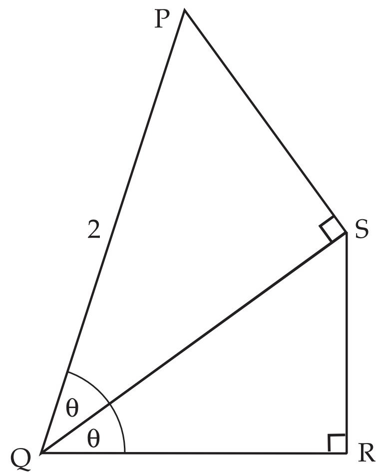
ans: proof 7 marks

•1 ss : eg start to complete square
•2 pd : complete process
•3 ic : complete proof
•4 ic : interpret vector (ie $\overrightarrow{BD}$)
•5 ss: state requirement for perpend.
•6 ic : complete proof
•7 ic : complete proof

•1 2 separate areas
•2 $\int_0^{\frac{\pi}{3}} (3 + 6\cos 2x)dx$
•3 $[3x + 3\sin 2x]$
•4 $\pi + \frac{3}{2}\sqrt{3}$
•5 $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (3 + 6\cos 2x)dx$
•6 $-3\sqrt{3} + \pi$
•7 $\pi + \frac{3}{2}\sqrt{3} + (-)\left(-3\sqrt{3} + \pi\right) = \ldots\ldots$

part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
a)	6	2.3.4	CN	B	2001HG q10
b)	4	2.3.4	CN	A	

In quadrilateral PQRS, PQ = 2 units, angle PQS = θ, angle SQR = θ, angle PSQ = $\frac{\pi}{2}$ and angle SRQ = $\frac{\pi}{2}$.



Show that

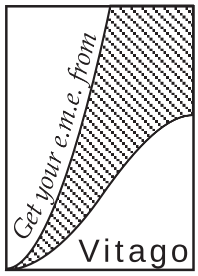
(a) the perimeter of of PQRS is $3 + 2\sin\theta + \sin 2\theta + \cos 2\theta$. **6**

(b) the area of PQRS can be expressed as $\sin 2\theta\left(2 - \sin^2\theta\right)$. **4**

Give 1 mark for each •	Illustrations for awarding each •
a **ans:** proof 6 marks	
•[1] ss : eg start to complete square	•[1] $PS = 2\sin\theta, QS = 2\cos\theta$
•[2] pd : complete process	•[2] $QR = 2\cos^2\theta$
•[3] ic : complete proof	•[3] $SR = 2\cos\theta\sin\theta$
•[4] ic : interpret vector (ie $\overrightarrow{BD}$)	•[4] $P = 2 + 2\sin\theta + 2\sin\theta\cos\theta + 2\cos^2\theta$
•[5] ss: state requirement for perpend.	•[5] $2\cos\theta\sin\theta = \sin 2\theta$
•[6] ic : complete proof	•[6] $2\cos^2\theta = 1 + \cos 2\theta$ *and complete*
b **ans:** proof 4 marks	
•[7] ic : complete proof	•[7] $\Delta PSQ: \frac{1}{2} 2\cos\theta 2\sin\theta$
•[8] ic : complete proof	•[8] $\Delta SQR: \frac{1}{2} 2\cos^2\theta 2\sin\theta\cos\theta$
•[9] ic : complete proof	•[9] $2\sin\theta\cos\theta\left[1 + \cos^2\theta\right]$
•[10] ic : complete proof	•[10] use $\cos^2\theta = 1 - \sin^2\theta$ *and* $2\sin\theta\cos\theta = \sin 2\theta$ *and complete*

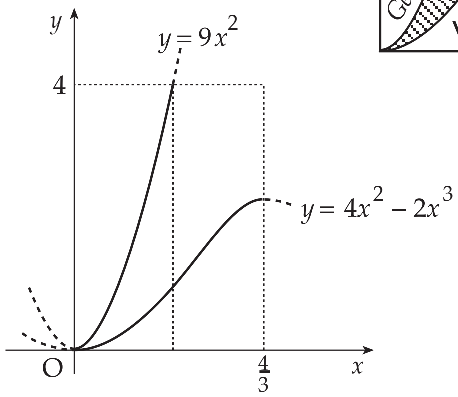
part	marks	Syllabus Code	Calc. Code (CN,CR,NC)	Grade (C, B, A)	Source
	10	2.2.7	CN	A	2001HG q11

The diagram shows the front of a packet of Vitago, a new vitamin preparation to provide early morning energy. The shaded region is red and the rest yellow.



The design was created by drawing the curves $y = 9x^2$ and $y = 4x^2 - 2x^3$. The edges of the packet are represented by the coordinate axes and the lines $x = \frac{4}{3}$ and $y = 4$.

Show that $\frac{10}{27}$ of the front of the packet is red. **10**



Give 1 mark for each • | Illustrations for awarding each •

ans: proof 10 marks

•1	ss : eg start to complete square	•1	$9x^2 = 4,\ \ x = \frac{2}{3}$
•2	pd : complete process	•2	$A = A_1 + rect - A_3$
•3	ic : complete proof	•3	$A_1 = \int_0^{\frac{2}{3}} \left(9x^2 - (4x^2 - 2x^3)\right)dx$
•4	ic : interpret vector (ie $\overrightarrow{BD}$)	•4	$\frac{5}{3}x^3 + \frac{1}{2}x^4$
•5	ss: state requirement for perpend.	•5	$A_1 = \frac{16}{27}$
•6	ic : complete proof	•6	$A_3 = \int_{\frac{2}{3}}^{\frac{4}{3}} \left(4x^2 - 2x^3\right)dx$
•7	ic : complete proof	•7	$\frac{4}{3}x^3 - \frac{1}{2}x^4$
•8	ic : complete proof	•8	$A_3 = \frac{104}{81}$
•9	ic : complete proof	•9	$A_1 + A_2 = \frac{160}{81}$
•10	ic : complete proof	•10	$red = \frac{\frac{160}{81}}{\frac{16}{3}} = \ldots \frac{10}{27}$